



Measurement of S11 & S12

(1) Apply input to Port 1 , terminate at Port 2 with a Matched Load $\therefore a_2 = 0$.

$$\therefore S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \text{reflection coefficient at Port 1 (matched load at Port 2).}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} = \text{Voltage transfer ratio Port 1 to Port 2 (matched load at Port 2).}$$

(2) Apply input to Port 2 , terminate at Port 1 with a matched load $\therefore a_1 = 0$.

$$\therefore S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} = \text{reflection coefficient at Port 2 (matched load at Port 1).}$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} = \text{transfer ratio Port 2 to Port 1 (matched load at Port 1).}$$

Note each S matrix element is in general complex eg

$$S_{i,j} = \left| S_{i,j} \right| e^{j\theta_{i,j}}$$

ratio of amplitudes phase difference of signals

$$\text{eg } S_{11} = 0.9 \angle -37 \equiv 0.9e^{j(-37)}$$

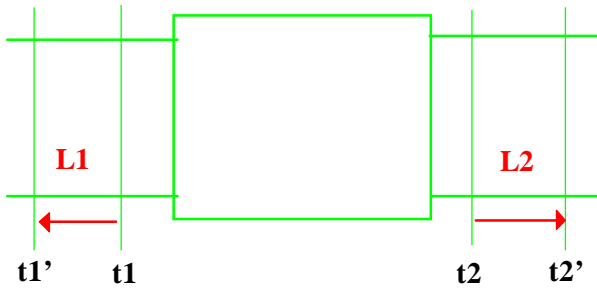
$$S_{21} = 4.0 \angle 127$$

Note S11,S22 are reflection coefficients \therefore can plot these on a Smith Chart.

S12,S21 are not reflection coefficients and are usually plotted on a Polar diagram.



Change of reference plane



Note positive signs for L1 & L2 are taken to be movements away from the device

Planes t1' and t2'

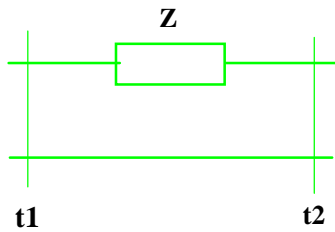
$$S_{11'} = S_{11} e^{-j2\beta L1} \quad S_{22'} = S_{22} e^{-j2\beta L2}$$

\uparrow \uparrow
 t1', t2' t1, t2

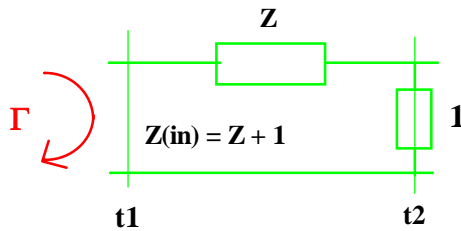
$$S_{21'} = S_{21} e^{-j\beta(L1 + L2)} \quad S_{12} = S_{12} e^{-j\beta(L1 + L2)}$$

S Matrix for simple elements

Series normalised impedance



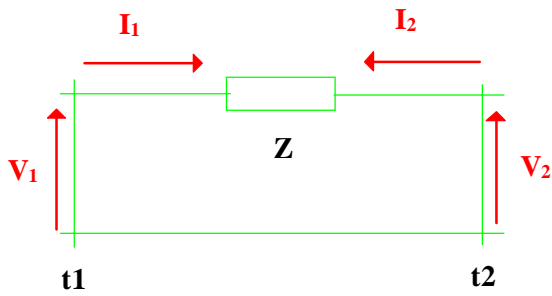
(1) Connect a matched load at t2 ie



$$\Gamma = \frac{Z_{(in)} - Z_0}{Z_{(in)} + Z_0} = \frac{Z_{(in)} - 1}{Z_{(in)} + 1} = \frac{z + 1 - 1}{z + 1 + 1} = \frac{z}{z + 2} = S_{11}$$

divide by Z_0
to normalise

$$S_{11} = S_{22} \text{ (By symmetry)} = \underline{\underline{\frac{z}{z + 2}}}$$



$$\left. \begin{array}{ll} V_1 = a_1 + b_1 & V_2 = a_2 + b_2 \\ I_1 = a_1 - b_1 & I_2 = a_2 - b_2 \end{array} \right\} \text{General}$$

Series connection $\therefore I_2 = -I_1$.

$$\therefore a_2 - b_2 = -(a_1 - b_1)$$

Matched load @ Port 2 $a_2 = 0$

$$\therefore b_2 = a_1 - b_1$$

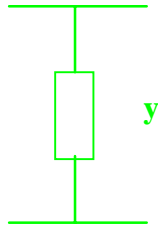
$$\therefore \left. \frac{b_2}{a_1} \right|_{a_2=0} = 1 - \frac{b_1}{a_1} = 1 - S_{11} \equiv S_{21}$$

$$S_{21} = 1 - S_{11} = 1 - \frac{z}{z+2} = \frac{2}{z+2} = S_{12} \text{ (by symmetry)}$$

$$\frac{z+2}{z+2} - \frac{z}{z+2} = \frac{\cancel{z} + 2}{z+2}$$



Shunt normalised Admittance



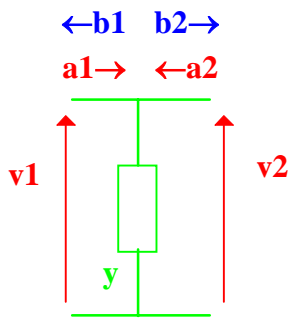
$$\text{Let } z = \frac{1}{y}$$

$$\therefore z_{in} = \frac{\frac{1}{y} \cdot 1}{\frac{1}{y} + 1} = \frac{\frac{1}{y}}{\frac{1}{y} + 1} \quad \text{x top \& bottom by } y$$

$$\frac{1}{y+1} = Z_{in}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{\frac{1}{y+1} - 1}{\frac{1}{y+1} + 1} \quad \text{x top \& bottom by } y+1$$

$$\frac{1 - y + 1}{1 + y + 1} = \frac{-y}{2 + y}$$



$$V_1 = V_2$$

$$\therefore a_2 + b_2 = a_1 + b_1$$

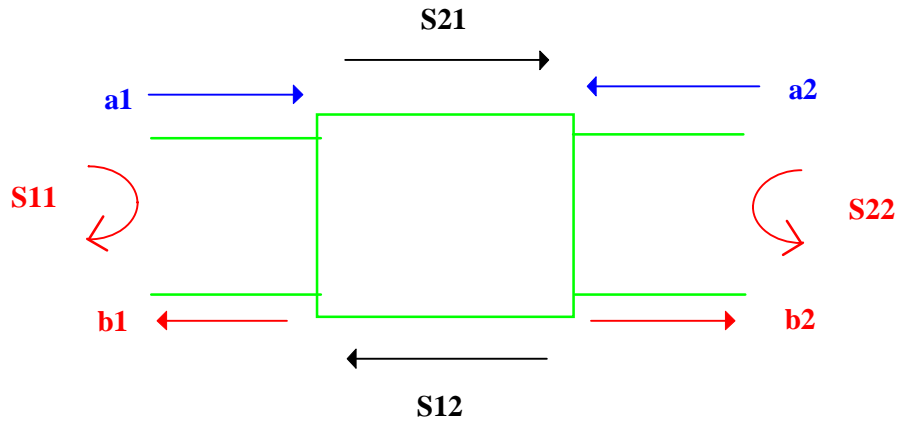
$$\text{matched load on Port 2 } \therefore a_2 = 0$$

$$S_{21} = \frac{b_2}{a_1} = \frac{a_1 + b_1}{a_1} = 1 + S_{11}$$

$$1 + \frac{-y}{2+y} = \frac{2+y}{2+y} + \frac{-y}{2+y} = \frac{2}{2+y} = S_{21}$$



S Parameters for a reciprocal, lossless network



At Port 1 Power in $|a_1|^2$

Power out $|b_1|^2$

\therefore Net power in $|a_1|^2 - |b_1|^2$



Reciprocal network

$$S_{21} = S_{12} \text{ (generally } S_{ij} = S_{ji} \text{)}$$

For a lossless network Power out = Power in

$$\sum_{\text{Port } i} |b_i|^2 = \sum_{\text{Port } i} |a_i|^2$$

$\Rightarrow [S]$ is a unitary matrix

$$\sum_{\text{Port } i} S_{i,s} \cdot S_{i,r}^* = \delta_{s,r} = 1 \quad \text{if } s=r$$

$$= 0 \quad \text{if } s \neq r$$

2port $i=1,2$ $s,r=1 \& 2$

$$S=1, r=1 \quad S_{11} \cdot S_{11}^* + S_{21} \cdot S_{21}^* = 1$$

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$\therefore |S_{21}| = \sqrt{1 - |S_{11}|^2}$$

$$S=2, r=2 \quad S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* = 1$$

$$|S_{12}|^2 + |S_{22}|^2 = 1$$

$$\therefore |S_{12}| = \sqrt{1 - |S_{22}|^2}$$

If the junction is reciprocal $\therefore |S_{12}| = |S_{21}| \quad \therefore |S_{22}| = |S_{11}|$



$$S = 1; r = 2 \quad S_{11} \cdot S_{12}^* + S_{21} \cdot S_{22}^* = 0$$

$$|S_{11}| e^{j\theta_{11}} \times |S_{12}| e^{-j\theta_{12}} + |S_{21}| e^{j\theta_{21}} \times |S_{22}| e^{-j\theta_{22}} = 0$$

$$\Rightarrow e^{j(\theta_{12} + \theta_{21})} = -e^{j(\theta_{11} + \theta_{22})}$$

$$\theta_{12} = \theta_{21} \quad (\text{reciprocal})$$

$$e^{j(2\theta_{12})} = -e^{j(\theta_{11} + \theta_{22})}$$

$$\cos 2\theta_{12} = -\cos(\theta_{11} + \theta_{22})$$

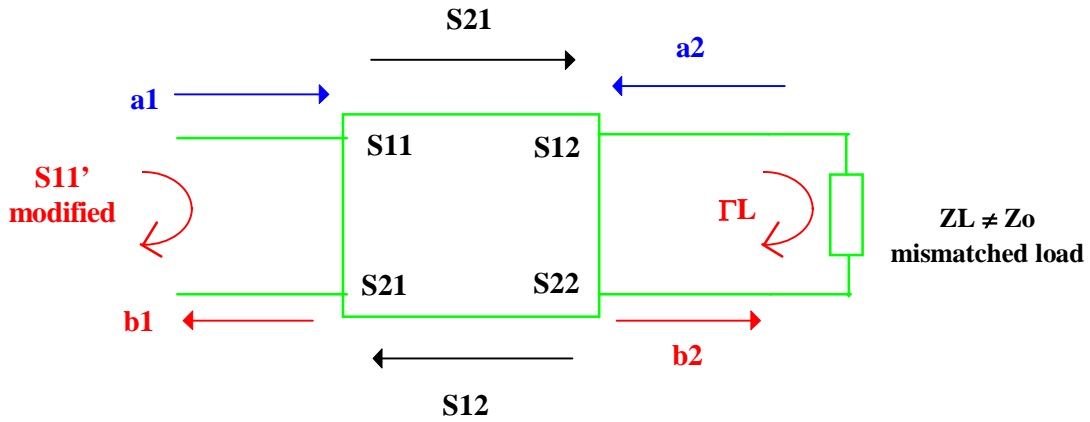
$$\sin 2\theta_{12} = -\sin(\theta_{11} + \theta_{22})$$

$$\Rightarrow \theta_{12} = \theta_{21} = \underline{\underline{\frac{1}{2}(\theta_{11} + \theta_{22}) + \frac{\pi}{2} \pm m\pi}} \quad m = 0, 1, 2, \dots$$

∴ For a reciprocal lossless 2 - port network we only need

$$|S_{11}|, \theta_{11} \text{ \& \ } \theta_{22}$$

Modified S Parameters - mismatched source & load



$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \quad \Gamma_L = \frac{a_2}{b_2} \quad \therefore a_2 = b_2 \cdot \Gamma_L$$

$$b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2 = S_{11} \cdot a_1 + S_{12} \cdot b_2 \cdot \Gamma_L$$

$$b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2 = S_{21} \cdot a_1 + S_{22} \cdot b_2 \cdot \Gamma_L$$

$$S_{11}' = \frac{b_1}{a_1} ; S_{21}' = \frac{b_2}{a_1}$$

$$S_{21}' = \frac{b_2}{a_1} = \frac{S_{21} \cdot a_1 + S_{22} \cdot b_2 \cdot \Gamma_L}{a_1} \quad \text{rearrange gives} \quad \frac{b_2}{a_1} - \frac{S_{22} \cdot b_2 \cdot \Gamma_L}{a_1} = \frac{S_{21} \cdot a_1}{a_1}$$

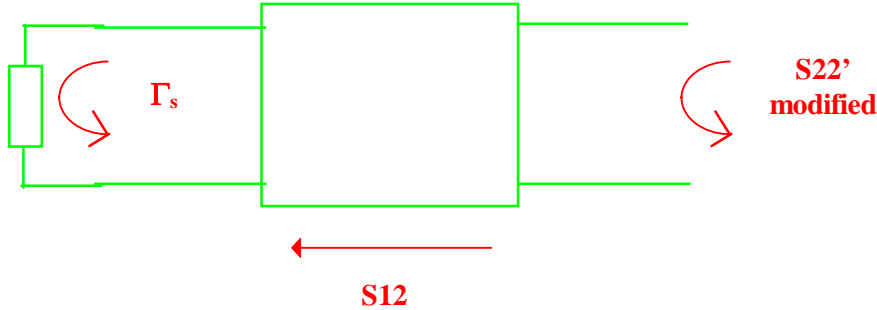
$$S_{21}' (1 - S_{22} \cdot \Gamma_L) = S_{21} \quad \therefore S_{21}' = \frac{S_{21}}{1 - S_{22} \cdot \Gamma_L}$$

$$b_1 = S_{11} \cdot a_1 + S_{12} \cdot \Gamma_L \cdot \left(\frac{S_{21}}{1 - S_{22} \cdot \Gamma_L} \right) \cdot a_1 \quad (\text{divide by } a_1 \text{ on both sides})$$

$$S_{11}' = S_{11} + \frac{S_{21} \cdot S_{12} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$



Mismatched Source

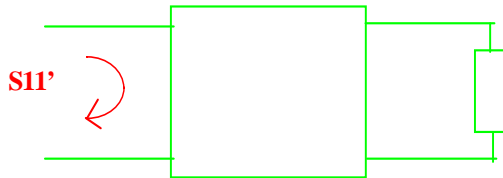


$$S_{22}' = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_s}{1 - S_{11} \cdot \Gamma_s}$$

$$S_{12}' = \frac{S_{12}}{1 - S_{11} \cdot \Gamma_s}$$

Circuit performance from S Parameters

Reflection characteristics:-



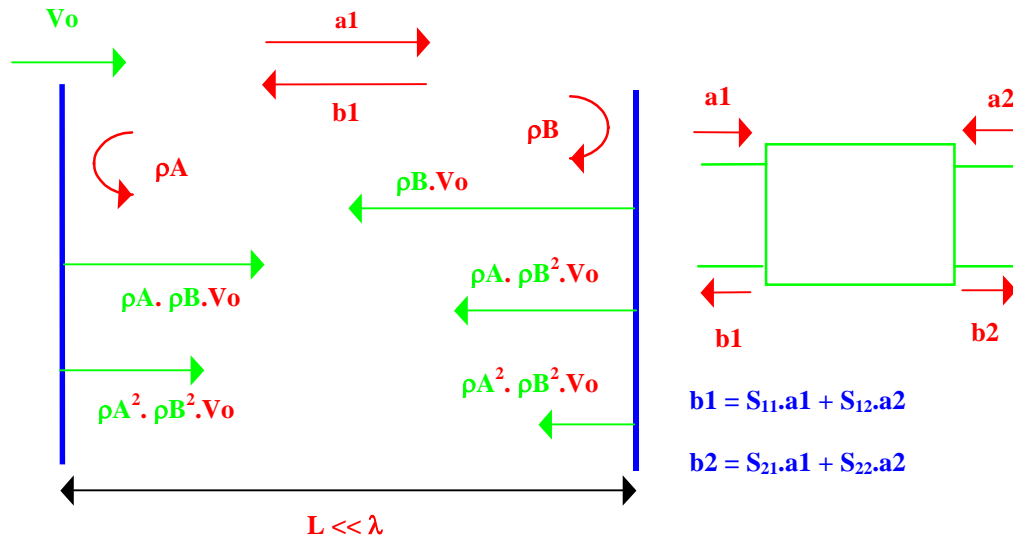
Input V.S.W.R = $\frac{1 + |S_{11}'|}{1 - |S_{11}'|}$ **modulus**

Return Loss (dB) = $10 \cdot \text{Log}_{10} \frac{\text{Power in}}{\text{Power reflected}}$

$$= 10 \cdot \text{Log}_{10} \frac{|a_1|^2}{|b_1|^2} = -20 \cdot \text{log}_{10} |S_{11}'|$$

$\frac{Z_{(in)}}{Z_o} = \frac{1 + \Gamma}{1 - \Gamma} = \frac{1 + S_{11}'}{1 - S_{11}'}$ **complex**

Multiple Reflection Effects



$$b1 = \rho_B.V_B + \rho_A.\rho_B^2 + \rho_A^2.\rho_B^2.V_o + \dots$$

$$= \rho_B.V_o[1 + \rho_A.\rho_B + (\rho_A.\rho_B)^2 + \dots]$$

$$b1 = \frac{\rho_B.V_o}{1 - \rho_A.\rho_B} \quad \rho_B = \frac{b1}{a1} \quad \therefore a1 = \frac{b1}{\rho_B}$$

$$\therefore a1 = \frac{V_o}{(1 - \rho_A.\rho_B)}$$

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