

Applied Mode Coordinate to Solve Dynamic Responses of Antenna Mechanic System

Li Pu* Sun Qinghong Chen Nan

(Department of Mechanical Engineering , Southeast University , Nanjing 210096 , China)

Abstract : Based on the Lagrange 's equation and the finite element method , this paper establishes the dynamic equation of a radar antenna mechanic system which is a high accuracy system and consists of two flexible bodies. Mode coordinates are used to reduce the orders of equation. Finally , the calculation method and engineering example are given when the rotational velocity of antenna is invariable and the wind velocity is 25 m/s. The error of antenna mechanic system can be estimated using the calculation results.

Key words : antenna , dynamics , mode coordinate

The measurement precision of radar is deeply influenced by the stiffness of antenna mechanic system , especially to the three dimensional radar which is a high accuracy system. In the typical calculation model of antenna mechanic system , the radar antenna and the antenna pedestal is calculated as an integrated structure , the rigid body degree between antenna and antenna pedestal is neglected. So it is not accurate enough to use the typical calculation model to calculate antenna mechanic error and antenna traction torque. In this paper , the dynamic equation of a radar antenna mechanic system which consists of two flexible bodies is established based on the Lagrange 's equation and the finite element method , the rigid body degree between antenna and antenna pedestal has not been neglected. Mode coordinate is used to reduce the orders of equation. Finally , the calculation method and engineering example are given when the angular velocity of antenna is invariable.

1 The Dynamic Equation of Radar Antenna Mechanic System

Fig.1 is the antenna mechanic system model. In general , we assume that the deformation of antenna is small enough , the damp of antenna can be neglected , and the foundation of antenna is stiffness enough. The antenna and the antenna pedestal are jointed by hinge with one rigid body degree. The coordinate frame $X^0Y^0Z^0$ is fixed at antenna pedestal. It is an Earth-fixed frame , where the vertical direction is given in Z^0 . The coordinate frame $X^1Y^1Z^1$ is fixed at

antenna , the coordinate origin is in hinge axis. The antenna and the antenna pedestal can be calculated separately using finite element method(FEM) , and the FEM node displacements of the antenna and the antenna pedestal are used as augmented coordinates. So we have augmented coordinates $q = [\theta \ a^0 \ a^1]^T$, where θ is the rotation angle vector of antenna , a^0 is the displacement matrix of antenna pedestal FEM nodes in $X^0Y^0Z^0$, a^1 is the displacement matrix of antenna FEM nodes in $X^1Y^1Z^1$.

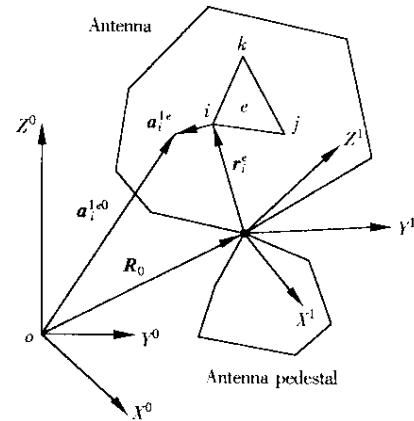


Fig.1 The model of radar antenna system

Using the augmented coordinates $q = [\theta \ a^0 \ a^1]^T$, the kinetic energy and the potential energy of antenna pedestal can be written as

$$V^0 = \frac{1}{2} q^T \bar{K}^0 q - q^T \bar{Q}^0 \quad (1)$$

$$T^0 = \frac{1}{2} \dot{q}^T \bar{M}^0 \dot{q} \quad (2)$$

In Earth-fixed frame $X^0Y^0Z^0$, the displacement

vector of node i in element e of the antenna shown in Fig. 1 is $\mathbf{a}_i^{1e0} = \mathbf{R}_0 + \mathbf{r}_i^e + \mathbf{a}_i^{1e}$, where \mathbf{r}_i^e is the position vector of FEM node i in frame $X^1Y^1Z^1$, \mathbf{a}_i^{1e} is the displacement vector of FEM node i in frame $X^1Y^1Z^1$.

The velocity vector of FEM node i in frame $X^0Y^0Z^0$ is $\dot{\mathbf{a}}_i^{1e0} = \dot{\mathbf{R}}_0 + \dot{\boldsymbol{\theta}} \times \mathbf{r}_i^e + \dot{\mathbf{a}}_i^{1e}$. Using the augmented coordinates, the potential energy of antenna can be written as

$$V^1 = \sum_{e=1}^{L_1} V^{1e} = \frac{1}{2} \mathbf{q}^T \bar{\mathbf{K}}^1 \mathbf{q} - \sum_{e=1}^{L_1} (\mathbf{r}^e)^T (\mathbf{A}^e)^T \mathbf{Q}^{1e} - \mathbf{q}^T \bar{\mathbf{Q}}^1 \quad (3)$$

where L_1 is the FEM element total number of antenna, the third term and the second term in Eq.(3) right hand side are the potential energy of external force (include external torque); \mathbf{r}^e is the position vector matrix of element nodes; \mathbf{A}^e is the transform matrix (it is the function of $\boldsymbol{\theta}$). It transforms the position and the displacement of FEM nodes in $X^1Y^1Z^1$ into the Earth-fixed frame $X^0Y^0Z^0$.

The kinetic energy of antenna in the Earth-fixed frame $X^0Y^0Z^0$ can be written as

$$T^{1e0} = \frac{1}{2} (\dot{\mathbf{a}}^{1e0})^T \mathbf{M}^{1e} \dot{\mathbf{a}}^{1e0}$$

$$\mathbf{M}^{1e} = \int_{V_e} \rho \mathbf{N}^T \mathbf{N} dV$$

where \mathbf{N} is the interpolation function. Using the augmented coordinates, the potential energy of antenna in the Earth-fixed frame $X^0Y^0Z^0$ can be written as

$$T^1 = \sum_{e=1}^{L_1} T^{1e0} = \frac{1}{2} \dot{\mathbf{q}}^T \bar{\mathbf{M}}^1 \dot{\mathbf{q}} \quad (4)$$

Using Eq.(1) – Eq.(4), the total kinetic energy and the total potential energy of antenna and antenna pedestal can be written as $T = T^0 + T^1$, $V = V^0 + V^1$.

According to Lagrange's equation $\frac{d}{dt} \left(\frac{\partial T - V}{\partial \dot{\mathbf{q}}_k} \right) - \frac{\partial (T - V)}{\partial \mathbf{q}_k} = 0$, we can deduce a $2n$ orders nonlinear differential equation

$$\begin{bmatrix} \mathbf{M}_{\theta\theta} & \mathbf{M}_{\theta 0} & \mathbf{M}_{\theta 1} \\ \mathbf{M}_{\theta 0}^T & \mathbf{M}_{00} & \mathbf{M}_{01} \\ \mathbf{M}_{\theta 1}^T & \mathbf{M}_{01}^T & \mathbf{M}_{11} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\theta}} \\ \ddot{\mathbf{a}}^0 \\ \ddot{\mathbf{a}}^1 \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathbf{M}_{\theta\theta}}{\partial t} & \frac{\partial \mathbf{M}_{\theta 0}}{\partial t} & \frac{\partial \mathbf{M}_{\theta 1}}{\partial t} \\ \frac{\partial \mathbf{M}_{\theta 0}^T}{\partial t} & \frac{\partial \mathbf{M}_{00}}{\partial t} & \frac{\partial \mathbf{M}_{01}}{\partial t} \\ \frac{\partial \mathbf{M}_{\theta 1}^T}{\partial t} & \frac{\partial \mathbf{M}_{01}^T}{\partial t} & \frac{\partial \mathbf{M}_{11}}{\partial t} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\mathbf{a}}^0 \\ \dot{\mathbf{a}}^1 \end{bmatrix} -$$

$$\begin{bmatrix} \frac{1}{2} \dot{\mathbf{q}}^T \frac{\partial \mathbf{M}}{\partial \boldsymbol{\theta}} \dot{\mathbf{q}} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{K}^0 & 0 \\ 0 & 0 & \mathbf{K}^1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{a}^0 \\ \mathbf{a}^1 \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_\theta + \sum_{e=1}^{L_1} [(\mathbf{r}^e)^T + (\mathbf{a}^{1e})^T] \left(\frac{\partial \mathbf{A}^e}{\partial \boldsymbol{\theta}} \right)^T \mathbf{Q}^{1e} \\ \mathbf{Q}^0 \\ \mathbf{Q}^1 \end{bmatrix} \quad (5)$$

Compared with \mathbf{r}^e , the deformation displacement \mathbf{a}^{1e} is very small and can be neglected. So the right side of Eq.(5) can be written as $\mathbf{Q}_\theta + \sum_{e=1}^{L_1} (\mathbf{r}^e)^T \left(\frac{\partial \mathbf{A}^e}{\partial \boldsymbol{\theta}} \right)^T \mathbf{Q}^{1e}$, where \mathbf{Q}_θ is antenna traction torque.

2 Modal Reduction and the Calculation Method

In general, the rotational velocity of antenna is required invariable. It means $\ddot{\boldsymbol{\theta}} = 0$, Eq.(5) can be written as

$$\begin{bmatrix} \mathbf{M}_{00} & \mathbf{M}_{01} \\ \mathbf{M}_{01}^T & \mathbf{M}_{11} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{a}}^0 \\ \ddot{\mathbf{a}}^1 \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathbf{M}_{00}}{\partial t} & \frac{\partial \mathbf{M}_{01}}{\partial t} \\ \frac{\partial \mathbf{M}_{01}^T}{\partial t} & \frac{\partial \mathbf{M}_{11}}{\partial t} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{a}}^0 \\ \dot{\mathbf{a}}^1 \end{bmatrix} + \begin{bmatrix} \mathbf{K}^0 & 0 \\ 0 & \mathbf{K}^1 \end{bmatrix} \begin{bmatrix} \mathbf{a}^0 \\ \mathbf{a}^1 \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \end{bmatrix} - \dot{\boldsymbol{\theta}} \begin{bmatrix} \frac{\partial \mathbf{M}_{00}^T}{\partial t} \\ \frac{\partial \mathbf{M}_{\theta 1}^T}{\partial t} \end{bmatrix} \quad (6a)$$

It can be abbreviated as

$$\ddot{\mathbf{M}}\ddot{\mathbf{A}} + \dot{\mathbf{C}}\dot{\mathbf{A}} + \mathbf{K}\mathbf{A} = \mathbf{F} \quad (6b)$$

where \mathbf{K} is constant matrix. The antenna traction torque can be written as

$$\mathbf{Q}_\theta = [\mathbf{M}_{\theta 0} \quad \mathbf{M}_{\theta 1}] \begin{bmatrix} \ddot{\mathbf{a}}^0 \\ \ddot{\mathbf{a}}^1 \end{bmatrix} + \left[\frac{\partial \mathbf{M}_{\theta\theta}}{\partial t} \quad \frac{\partial \mathbf{M}_{\theta 0}}{\partial t} \quad \frac{\partial \mathbf{M}_{\theta 1}}{\partial t} \right] \begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\mathbf{a}}^0 \\ \dot{\mathbf{a}}^1 \end{bmatrix} - \frac{1}{2} [\dot{\boldsymbol{\theta}} \quad \dot{\mathbf{a}}^0 \quad \dot{\mathbf{a}}^1] \frac{\partial \mathbf{M}}{\partial \boldsymbol{\theta}} \begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\mathbf{a}}^0 \\ \dot{\mathbf{a}}^1 \end{bmatrix} - \sum_{e=1}^{L_1} (\mathbf{r}^e)^T \left(\frac{\partial \mathbf{A}^e}{\partial \boldsymbol{\theta}} \right)^T \mathbf{Q}^{1e} \quad (7)$$

The Eq.(5) is established using FEM, so the order of Eq.(6b) is very large. It is too much calculation work to solve Eq.(6b) directly. But the modal function of antenna and antenna pedestal can be calculated firstly using FEM, then the order of Eq.(6b) can be reduced greatly using mode coordinate. $\begin{bmatrix} \mathbf{a}^0 \\ \mathbf{a}^1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\phi}^0 & 0 \\ 0 & \boldsymbol{\phi}^1 \end{bmatrix} \begin{bmatrix} \mathbf{X}^0 \\ \mathbf{X}^1 \end{bmatrix} = \boldsymbol{\phi}\mathbf{X}$, where \mathbf{X} is modal coordinate, Eq.(6b) can be written as

$$\boldsymbol{\phi}^T \mathbf{M} \boldsymbol{\phi} \ddot{\mathbf{X}} + \boldsymbol{\phi}^T \mathbf{C} \boldsymbol{\phi} \dot{\mathbf{X}} + \boldsymbol{\phi}^T \mathbf{K} \boldsymbol{\phi} \mathbf{X} = \boldsymbol{\phi}^T \mathbf{F} \quad (8)$$

The order of Eq.(8) is small enough to be calculated.

One period of antenna rotation can be divided into n intervals and every interval is very equal. In every interval, the change of θ is very small and can be neglected, θ can be treated as a constant. In arbitrary m th interval, the derivative

$$\frac{\partial \mathbf{M}}{\partial t} = \frac{[\mathbf{M}(m\Delta\theta) - \mathbf{M}((m-1)\Delta\theta)]}{\Delta t}$$

$$\frac{\partial \mathbf{M}}{\partial \theta} = \frac{[\mathbf{M}(m\Delta\theta) - \mathbf{M}((m-1)\Delta\theta)]}{\Delta\theta}$$

The external force \mathbf{F} can also be treated using this method in arbitrary m th interval. Thus Eq.(6b) is a linear differential equation in every interval, it can be solved easily. The process of solution is as follows.

First, X_0 and \dot{X}_0 are set to zero in 0 degree of antenna rotation angle, they are the initial value of interval 1th. Matrix $\mathbf{M}, \mathbf{C}, \mathbf{K}, \mathbf{F}$ can be established using FEM, the value $X_1, \dot{X}_1, \ddot{X}_1$ can be calculated in interval 1th.

Second, X_1 and \dot{X}_1 are set to the initial value of interval 2th. Matrix $\mathbf{M}, \mathbf{C}, \mathbf{F}$ should be built over again and the value $X_2, \dot{X}_2, \ddot{X}_2$ can be calculated in interval 2th.

Third, X_2 and \dot{X}_2 are set to the initial value of interval 3th. Matrix $\mathbf{M}, \mathbf{C}, \mathbf{F}$ should be built over again and the value $X_3, \dot{X}_3, \ddot{X}_3$ can be calculated in interval 2th.

.....

The last step, X_{n-1} and \dot{X}_{n-1} are set to the initial value of interval n th. Matrix $\mathbf{M}, \mathbf{C}, \mathbf{F}$ should be built over again and the value $X_n, \dot{X}_n, \ddot{X}_n$ can be calculated in interval n th. $X_n, \dot{X}_n, \ddot{X}_n$ are on the 0 degree of antenna rotation angle.

Compare X_0, \dot{X}_0 with X_n, \dot{X}_n , if the error between X_0, \dot{X}_0 and X_n, \dot{X}_n is too big, substitute X_n, \dot{X}_n for X_0, \dot{X}_0 , come back to the first step, calculate again, till the error is small enough.

According to X_n, \dot{X}_n , the antenna traction torque can be calculated using Eq.(7).

3 Example

Fig.2 is a radar antenna system. Its rotation per minute is six circles. We assume that wind speed is 20 m/s, and the wind direction is invariable. Now we calculate the antenna traction torque and the displacement of point A in antenna when the antenna rotation per minute is kept on six circles.

Firstly one period of antenna rotation is divided into 360 intervals and every interval is equal. In every

interval, the mass matrix, stiffness matrix and relevant matrix are formed. Secondly a seven orders equation can be built in every interval when the first three modals of antenna and antenna pedestal are used. Thirdly, the equation has been solved using above-mentioned method.

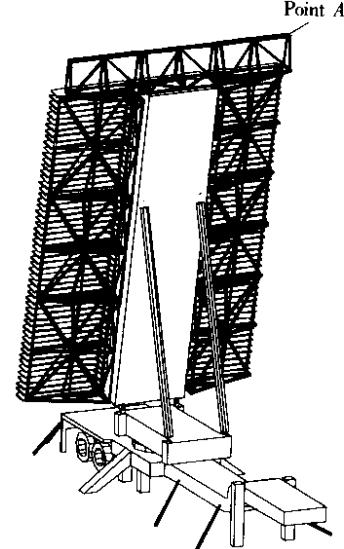


Fig.2 The radar antenna system in calculation example

The calculation results of first twenty seconds have been drawn in Fig.3 and Fig.4. Fig.3 is the displacement of point A in $X^0Y^0Z^0$. Fig.4 is the antenna traction torque. Because the rotation radius of point A is 3.5 m, the deformation displacement of point A is too small to be observed versus the rigid displacement of point A, so the deformation displacement of point A is amplified 500 times in Fig.3. And in Fig.4, the velocity and the acceleration of point A in Eq.(7) are also amplified 500 times.

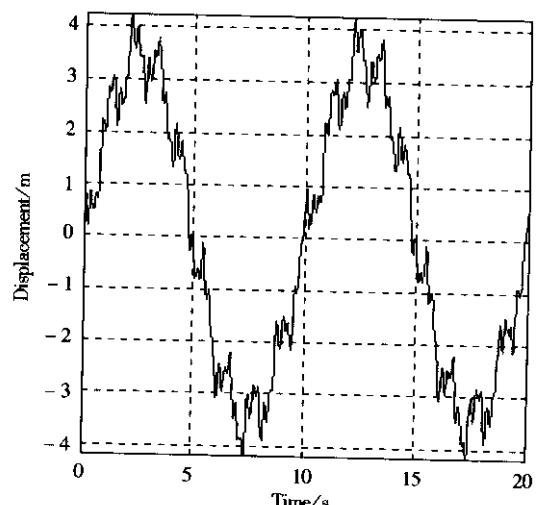


Fig.3 The displacement of point A

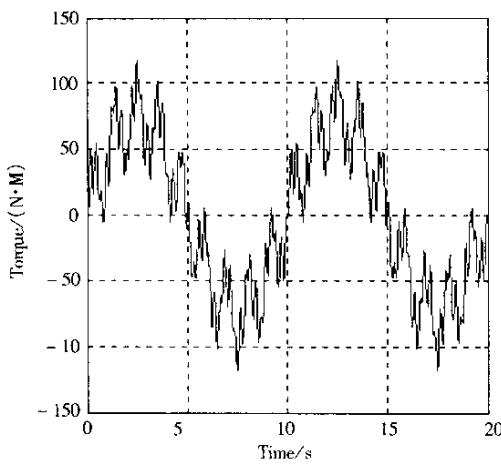


Fig.4 The antenna traction torque

4 Conclusion

In this paper, the dynamic equation of a radar antenna mechanic system which consists of two flexible bodies is established based on the Lagrange's equation and the finite element method, mode coordinate is used to reduce the orders of equation. Finally, the calculation method and engineering example are given when the

angular velocity of antenna is invariable. The equations in this paper are built on the small deformation assumption. Large deformation is not allowed for radar antenna, so the assumption is rational.

Because mode coordinate is used to reduce the orders of equation, the solution efficiency is very higher than the direct solution.

References

- [1] Zhang D J, Liu C Q, Huston R L. On the dynamics of an arbitrary flexible body with large overall motion: an integrated approach[J]. *Mech Struct & Mach*, 1995, 23(3): 419–438.
- [2] Hairing W J. New formulation for flexible beams under large overall plane motion[J]. *J Guid Cont and Dyn*, 1994, 17: 76–83.
- [3] Kane T R, Ryan R R, Banerjee A K. Dynamics of a cantilever beam attached to a moving base[J]. *J Guid Cont and Dyn*, 1987, 10: 139–151.
- [4] Hanagud S, Sarkar S. Problem of the dynamics of a cantilever beam attached to a moving base[J]. *J Guid Cont and Dyn*, 1989, 12: 438–441.
- [5] Banerjee A K, Kane T R. Dynamics of a plate in large overall motion[J]. *J Appl Mech*, 1989, 56: 887–892.

应用模态坐标求解天线机械系统动力学响应

李 普 孙庆鸿 陈 南

(东南大学机械工程系,南京 210096)

摘要 应用 Lagrange 方程和有限元法建立了由 2 个弹性体组成的有高精度要求的某型雷达天线机械系统动力学方程,并应用模态坐标进行了降阶,大大降低了方程自由度,减轻了计算量,给出了天线匀速旋转时的方程解法,得到了天线机械系统在 25 m/s 风速下保持匀速运转时的位移响应和所需的驱动力矩. 为天线机械系统误差确定和驱动系统设计提供了依据.

关键词 天线, 动力学, 模态坐标

中图分类号 TN03; O39

如何学习天线设计

天线设计理论晦涩高深，让许多工程师望而却步，然而实际工程或实际工作中在设计天线时却很少用到这些高深晦涩的理论。实际上，我们只需要懂得最基本的天线和射频基础知识，借助于 HFSS、CST 软件或者测试仪器就可以设计出工作性能良好的各类天线。

易迪拓培训(www.edatop.com)专注于微波射频和天线设计人才的培养，推出了一系列天线设计培训视频课程。我们的视频培训课程，化繁为简，直观易学，可以帮助您快速学习掌握天线设计的真谛，让天线设计不再难…



HFSS 天线设计培训课程套装

套装包含 6 门视频课程和 1 本图书，课程从基础讲起，内容由浅入深，理论介绍和实际操作讲解相结合，全面系统的讲解了 HFSS 天线设计的全过程。是国内最全面、最专业的 HFSS 天线设计课程，可以帮助你快速学习掌握如何使用 HFSS 软件进行天线设计，让天线设计不再难…

课程网址: <http://www.edatop.com/peixun/hfss/122.html>

CST 天线设计视频培训课程套装

套装包含 5 门视频培训课程，由经验丰富的专家授课，旨在帮助您从零开始，全面系统地学习掌握 CST 微波工作室的功能应用和使用 CST 微波工作室进行天线设计实际过程和具体操作。视频课程，边操作边讲解，直观易学；购买套装同时赠送 3 个月在线答疑，帮您解答学习中遇到的问题，让您学习无忧。

详情浏览: <http://www.edatop.com/peixun/cst/127.html>



13.56MHz NFC/RFID 线圈天线设计培训课程套装

套装包含 4 门视频培训课程，培训将 13.56MHz 线圈天线设计原理和仿真设计实践相结合，全面系统地讲解了 13.56MHz 线圈天线的工作原理、设计方法、设计考量以及使用 HFSS 和 CST 仿真分析线圈天线的具体操作，同时还介绍了 13.56MHz 线圈天线匹配电路的设计和调试。通过该套课程的学习，可以帮助您快速学习掌握 13.56MHz 线圈天线及其匹配电路的原理、设计和调试…

详情浏览: <http://www.edatop.com/peixun/antenna/116.html>



关于易迪拓培训:

易迪拓培训(www.edatop.com)由数名来自于研发第一线的资深工程师发起成立,一直致力于专注于微波、射频、天线设计研发人才的培养;后于 2006 年整合合并微波 EDA 网(www.mweda.com),现已发展成为国内最大的微波射频和天线设计人才培养基地,成功推出多套微波射频以及天线设计经典培训课程和 **ADS**、**HFSS** 等专业软件使用培训课程,广受客户好评;并先后与人民邮电出版社、电子工业出版社合作出版了多本专业图书,帮助数万名工程师提升了专业技术能力。客户遍布中兴通讯、研通高频、埃威航电、国人通信等多家国内知名公司,以及台湾工业技术研究院、永业科技、全一电子等多家台湾地区企业。

我们的课程优势:

- ※ 成立于 2004 年, 10 多年丰富的行业经验
- ※ 一直专注于微波射频和天线设计工程师的培养, 更了解该行业对人才的要求
- ※ 视频课程、既能达到了现场培训的效果, 又能免除您舟车劳顿的辛苦, 学习工作两不误
- ※ 经验丰富的一线资深工程师主讲, 结合实际工程案例, 直观、实用、易学

联系我们:

- ※ 易迪拓培训官网: <http://www.edatop.com>
- ※ 微波 EDA 网: <http://www.mweda.com>
- ※ 官方淘宝店: <http://shop36920890.taobao.com>