Appendix B
Digital Modulation

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- **Appendix B Digital Modulation**
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Digital Modulation

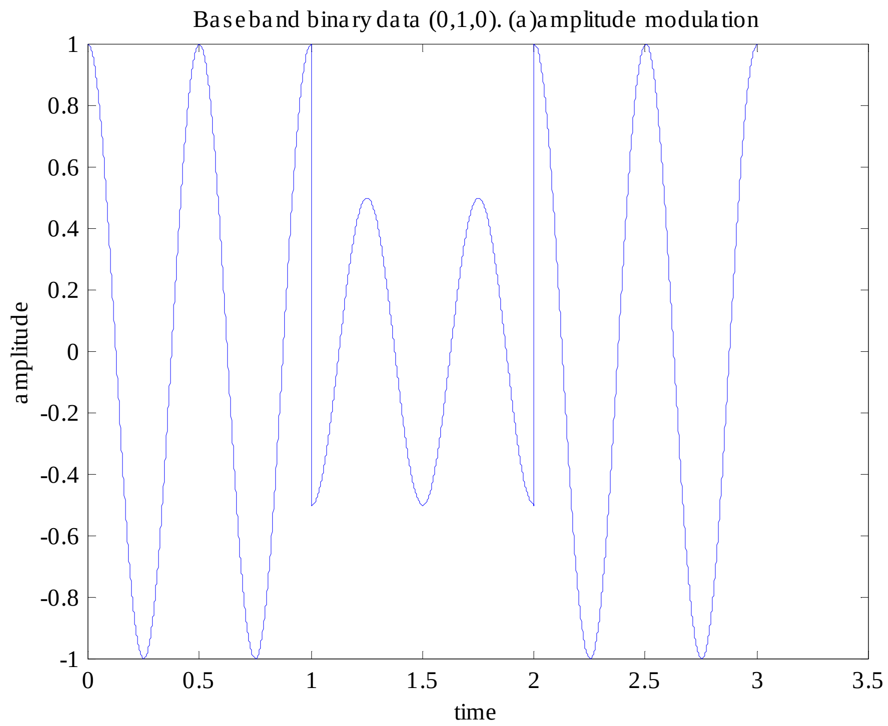
- **Cosine-wave modulation waveform**
  - $v(t) = A\cos(2\pi f_c t + \theta)$

- **There are three basic ways of modulating a carrier**
  - **Amplitude** A
  - **Frequency** f_c
  - **Phase** θ

- **Any one of the three quantities, frequency f_c, amplitude A or phase θ, may be varied in accordance with the information-carrying, or modulating signal.**

Digital Modulation

- **Baseband binary data (0,1,0)**
  - **Amplitude modulation**
  - **We represent a 0 by a high-amplitude signal**
  - **We represent a 1 by a low-amplitude signal**



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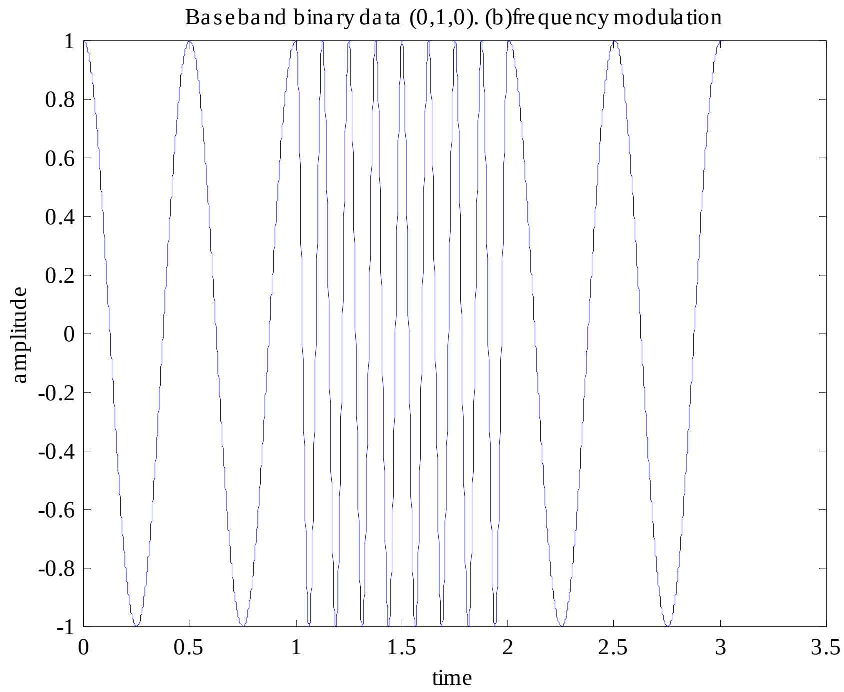
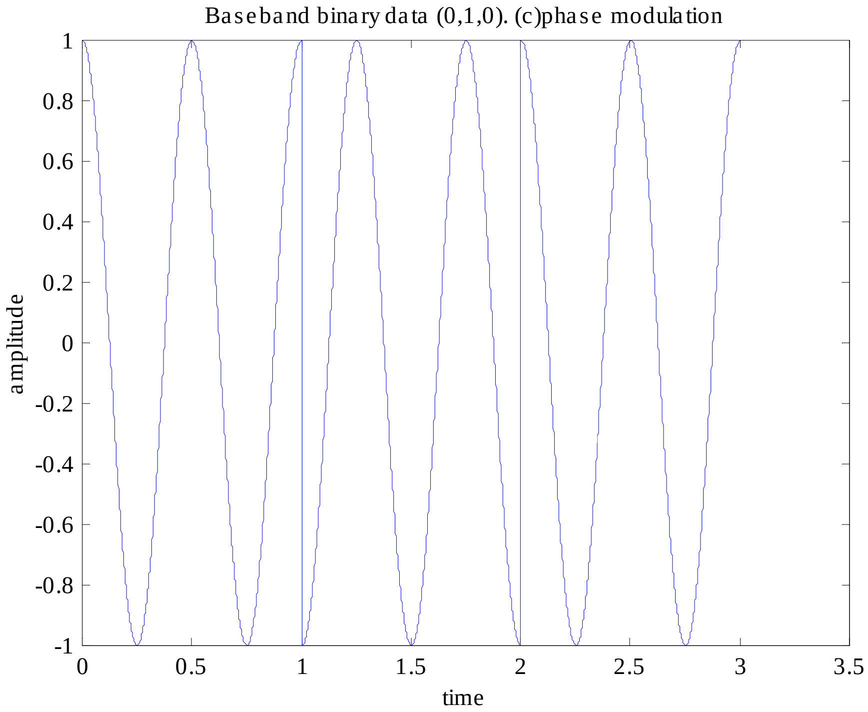
Digital Modulation

- **Baseband binary data (0,1,0)**
  - **Frequency modulation**
  - **We represent a 0 by a low frequency signal**
  - **We represent a 1 by a high-frequency signal**



Digital Modulation

- **Baseband binary data (0,1,0)**
  - **Phase modulation**
  - **We represent a 0 by a signal with phase 0**π
  - **We represent a 1 by a signal with phase**



Digital Modulation

- Digital modulation includes
  - Amplitude Shift Keying
  - Phase Shift Keying
  - Frequency Shift Keying
  - Quadrature Amplitude Modulation
- Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM) are the popular types of modulation in combined with OFDM.
- In This Appendix, we just consider
  - Phase Shift Keying
    - Binary Phase Shift Keying
    - Quadriphase shift Keying
    - M-ary Phase Shift Keying
    - Differential Phase Shift Keying
  - Quadrature Amplitude Modulation

B.1 Phase Shift Keying

- The information is carried by phase of modulated carrier.
- Coherent phase shift keying
  - Coherent means doing with known carrier phase information
- Non-coherent coherent shift keying
  - Non-coherent means doing without carrier phase information.

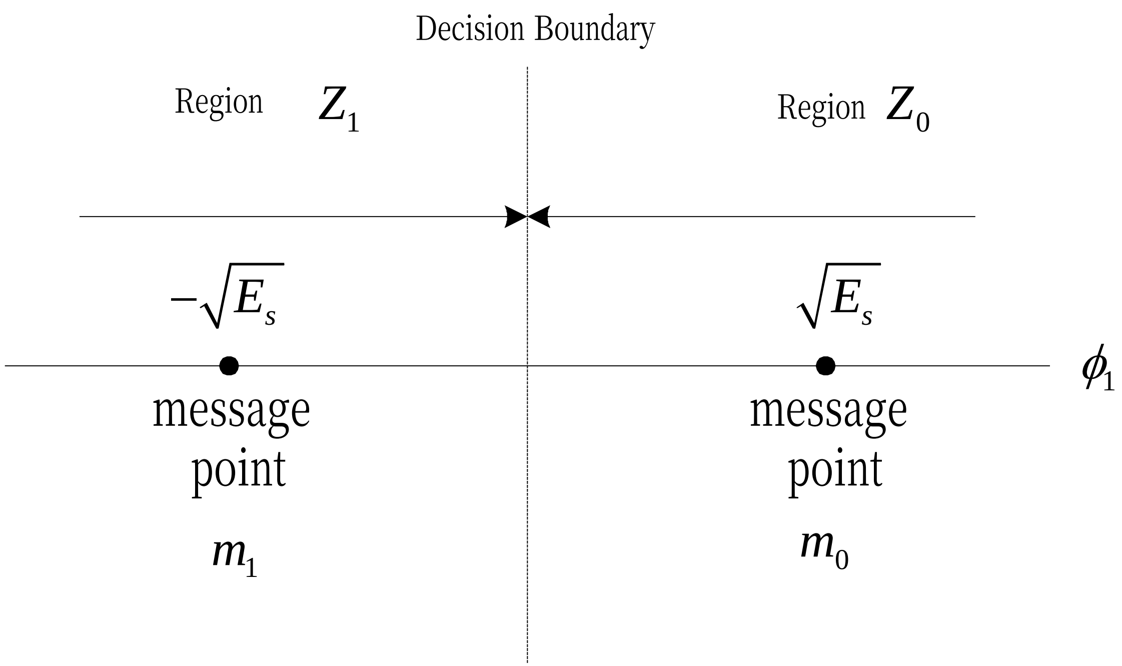
B.1.1 Binary Phase Shift Keying

- **In binary phase shift keying the phase of a carrier is switched between two values according to the to possible messages $s_0(t)$ and $s_1(t)$.**
- $$\begin{cases} s_0(t) = A\cos(2\pi f_c t + \theta_0) \\ s_1(t) = A\cos(2\pi f_c t + \theta_1) \end{cases}$$

  where θ_0 and θ_1 are constant phase shift. The two phases are usually separated by π radians.

B.1.1 Binary Phase Shift Keying

☞ **Signal constellation**



B.1.2 Quadriphase Shift Keying

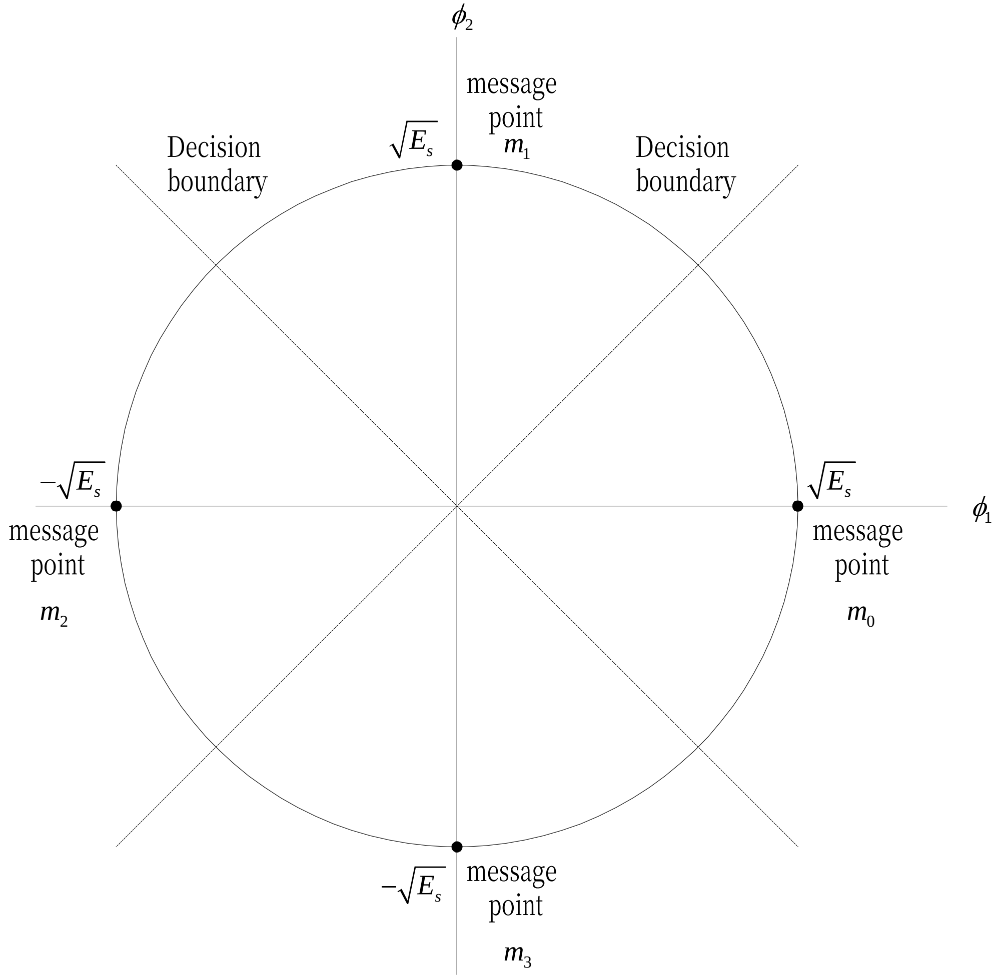
☞ In binary phase shift keying the phase of a carrier is switched between four values according to the to possible messages $s_0(t)$, $s_1(t)$, $s_2(t)$ and $s_3(t)$.

☞
$$\begin{cases} s_0(t) = A\cos(2\pi f_c t + \theta_0) \\ s_1(t) = A\cos(2\pi f_c t + \theta_1) \\ s_2(t) = A\cos(2\pi f_c t + \theta_2) \\ s_3(t) = A\cos(2\pi f_c t + \theta_3) \end{cases}$$

where θ_0, θ_1, θ_2 and θ_3 are constant phase shift. The four phases are usually separated by $\frac{\pi}{2}$ radians.

B.1.2 Quadriphase Shift Keying

☞ Signal constellation



B.1.3 M-ary Phase Shift Keying

- **Consider *M*-ary Phase Shift Keying (MPSK) for which the signal set is**

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{2\pi(i-1)}{M}\right) \quad 0 \le t \le T_s, \quad i = 1, 2, \ldots, M \quad ,$$

where E_s is the signal energy per symbol, T_s is the symbol duration, and f_c is the carrier frequency. This phase of the carrier takes on one of the *M* possible values, namely,

$\theta_i = 2(i-1)\pi/M$ **, where** $i = 1, 2, \ldots, M$ **.**

B.1.3 M-ary Phase Shift Keying

☞ **It is easier to use trigonometric identities as**

$$s_i(t)=\sqrt{E_s}\left[\cos\frac{2\pi(i-1)}{M}\sqrt{\frac{2}{T_s}}\cos 2\pi f_c t-\sin\frac{2\pi(i-1)}{M}\sqrt{\frac{2}{T_s}}\sin 2\pi f_c t\right]$$

$$=\sqrt{E_s}\left[\cos\frac{2\pi(i-1)}{M}\phi_1(t)-\sin\frac{2\pi(i-1)}{M}\phi_2(t)\right]\qquad 0\le t\le T_s,\quad i=1,2,\dots,M.$$

where it follows that

$$\phi_1(t)=\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),\qquad 0\le t\le T_s$$

$$\phi_2(t)=\sqrt{\frac{2}{T_s}}\sin(2\pi f_c t),\qquad 0\le t\le T_s$$

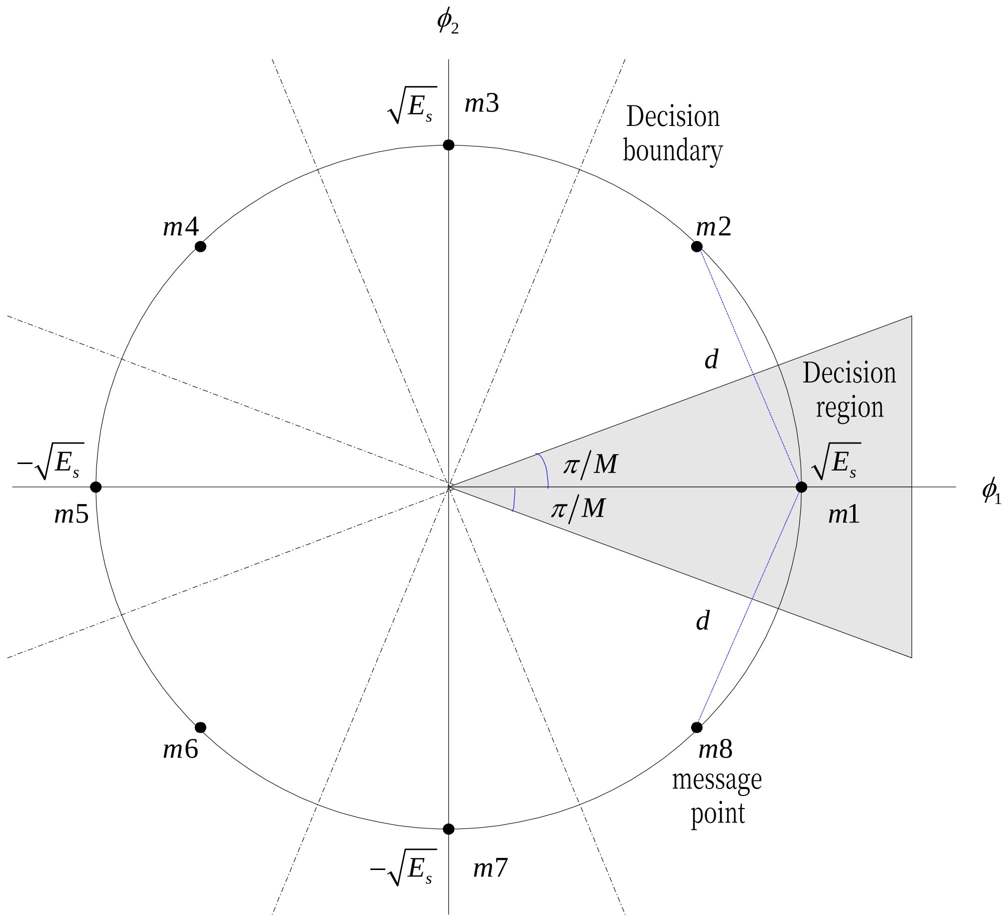
B.1.3 M-ary Phase Shift Keying

- **That is to say, each $s_i(t)$ may be expanded in terms of the same two orthogonal basis functions $\phi_1(t)$ and $\phi_2(t)$.**
- **The signal constellation of *M*-ary PSK is therefore two-dimensional.**

B.1.3 M-ary Phase Shift Keying

Signal constellation

- For example, 8-PSK



B.1.3 M-ary Phase Shift Keying

- **Signal constellation**
  - **The signal-space diagram is circularly symmetric.**
  - **The best decision strategy chooses the signal point in the signal space closest in Euclidean distance to the received data point.**
  - **The Euclidean distance of each of adjacent two points is**

$$d = 2\sqrt{E_s}\sin\left(\frac{\pi}{M}\right)$$

  - **The average probability of symbol error for coherent *M*-ary PSK is**

$$p_e \simeq erfc\left(\sqrt{\frac{E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right) ,$$

where it is assumed that $M \geq 4$.

B.1.3 M-ary Phase Shift Keying

- M-ary PSK
  - BPSK

$$p_{e,BPSK} = \frac{1}{2} erfc\left(\sqrt{\gamma}\right)$$

  - QPSK with Gray code

$$p_{e,QPSK} = \frac{1}{2} erfc\left(\sqrt{\gamma}\right)$$

  - M-ary PSK

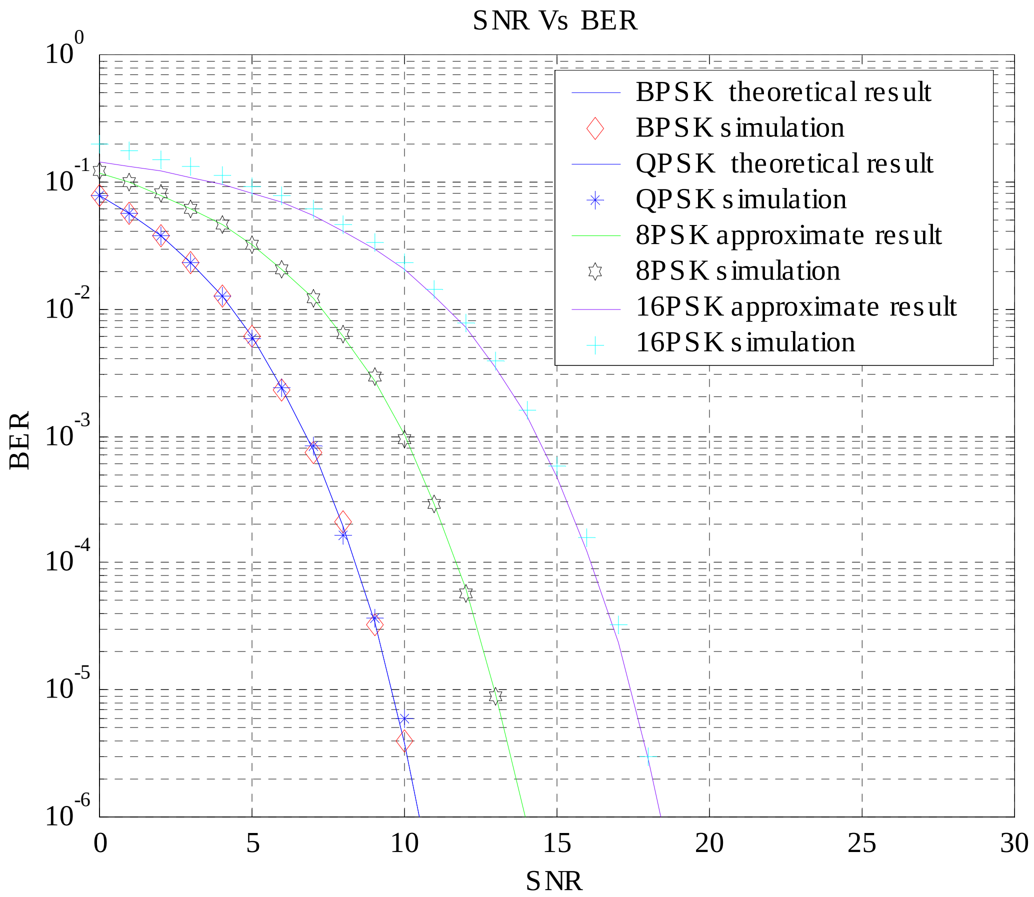
$$p_e \simeq erfc\left(\sqrt{\frac{E_s}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$$

where

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-z^2) dz$$

B.1.3 M-ary Phase Shift Keying

☞ **BER versus SNR curves for the OFDM system in AWGN channel using BPSK, QPSK, 8PSK,16-PSK .**



B.1.4 Differential Phase Shift Keying

- **Two basic operations of DPSK**
  - **Differential encoding of the input binary wave**
  - **Phase shift keying**
- **A differentially encoded phase-modulated signal allows non-coherent demodulation that does not require the estimation of the carrier phase.**
- **The performance of DPSK is 3dB better than that of PSK.**

B.2 Quadrature Amplitude Modulation

- **Quadrature Amplitude Modulation (QAM) is the most popular type of modulation in combined with OFDM.**
- **Rectangular constellations are especially easy to be implemented as they can be split into independent pulse amplitude modulation (PAM) components for both the in-phase and the quadrature part.**

B.2 Quadrature Amplitude Modulation

- **The transmitted *M*-ary QAM signal for symbol *n* can be expressed as**

$$s_n(t)=\sqrt{\frac{2E}{T}}a_n\cos(2\pi f_c t)-\sqrt{\frac{2E}{T}}b_n\sin(2\pi f_c t),\quad 0\le t\le T,\quad n=0,\pm1,\pm2,....,$$

where 2*E* is the energy of the signal with the lowest amplitude, a_n **and** b_n **are amplitudes taking on the values**

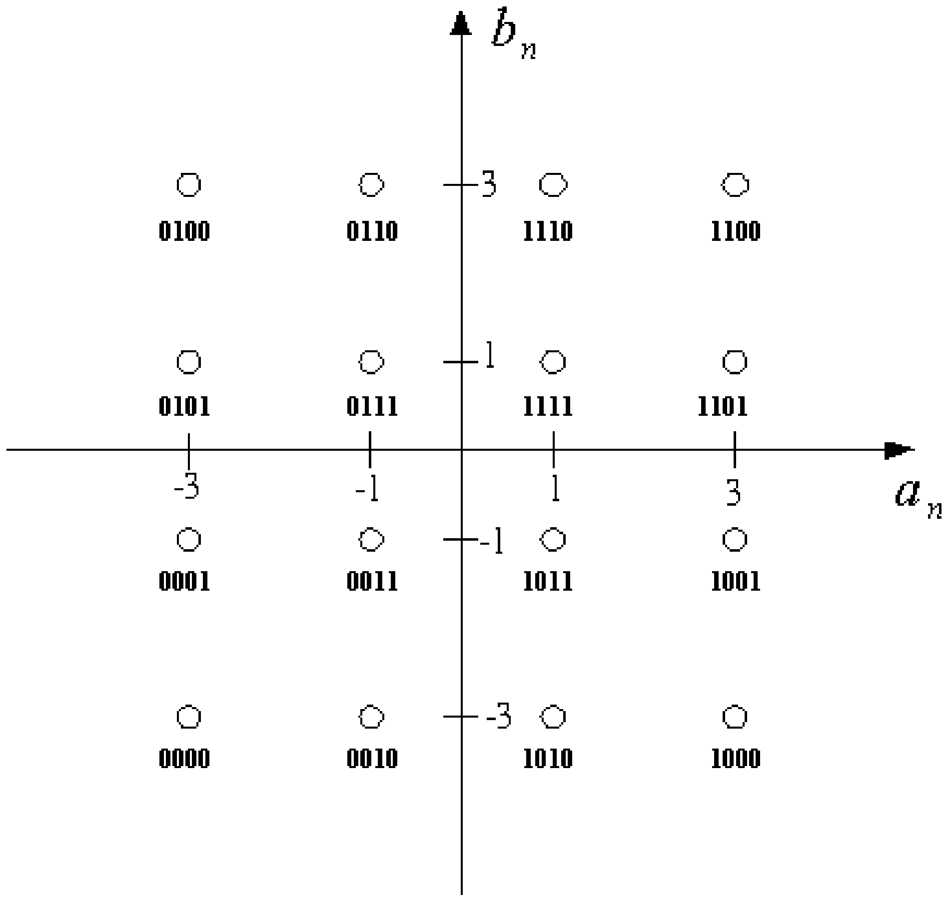
$$a_n, b_n = \pm a, \pm 3a, \ldots, \pm\left(\log_2 M - 1\right)a.$$

- **Note that *M* is assumed to be a power of *4*.**
- **The parameter *a* can be related to the average signal energy by** $a=\sqrt{\frac{3E_{av}}{2(M-1)}}$ **.**

B.2 Quadrature Amplitude Modulation

Signal constellation

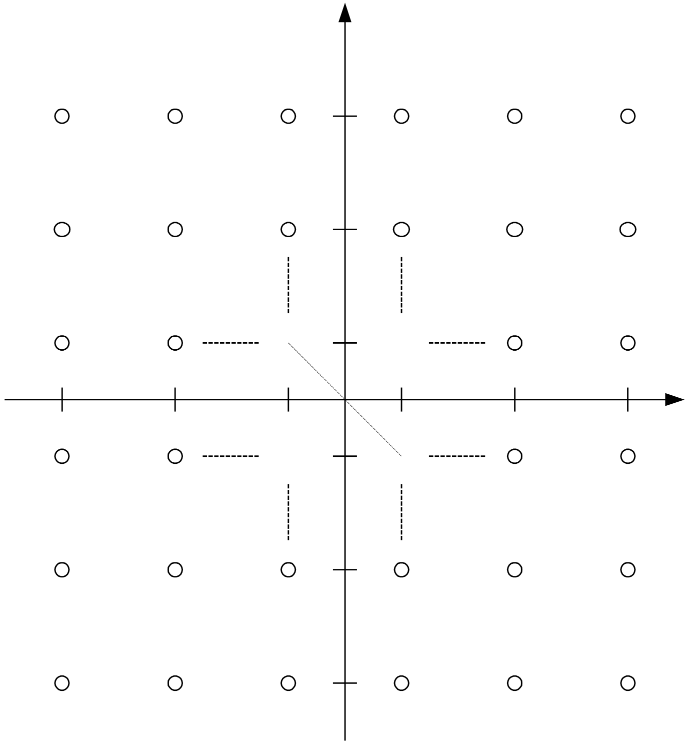
- The rectangular constellations of 16-QAM with Gray code.



B.2 Quadrature Amplitude Modulation

☞ **The probability of symbol error for *M*-ary QAM is**

$$p_{e,M-QAM} = \frac{1}{\log_2 M}\left\{1-\frac{1}{M}\left[\left(\sqrt{M}-2\right)^2 \cdot p(c|\mathrm{I}) + 4\left(\sqrt{M}-2\right)p(c|\mathrm{II}) + 4p(c|\mathrm{III})\right]\right\}$$



$$a = \sqrt{\frac{3E_s}{2(M-1)}}$$

$$p(c|\mathrm{I}) = \left[1-2Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]^2$$

$$p(c|\mathrm{II}) = \left[1-2Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]\left[1-Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]$$

$$p(c|\mathrm{III}) = \left[1-Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]^2$$

B.2 Quadrature Amplitude Modulation

- BER versus SNR curves for the OFDM system in AWGN channel using BPSK/QPSK, 16QAM, 64QAM, 256QAM.