Chapter 2
The Basic Principles of OFDM

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2 The Basic Principles of OFDM [1-7]

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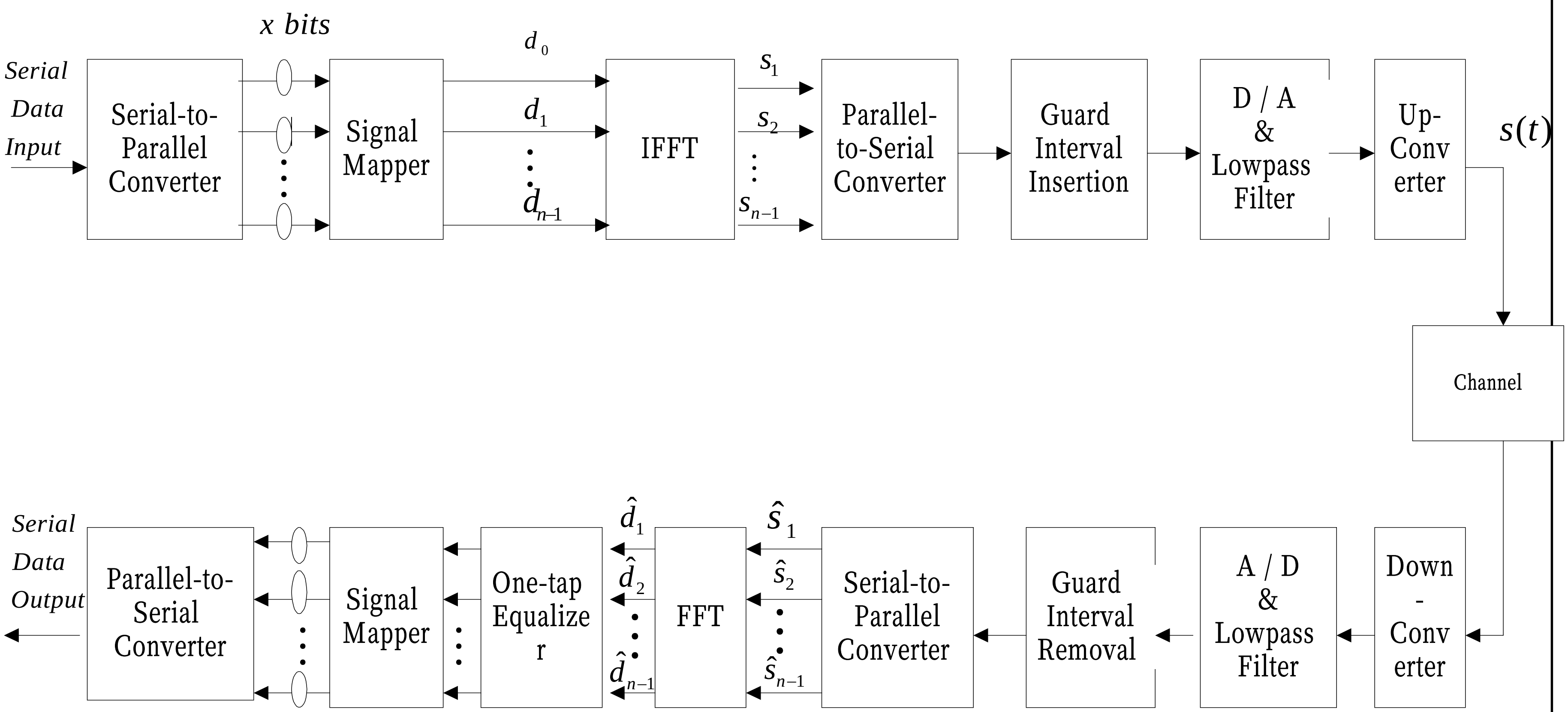
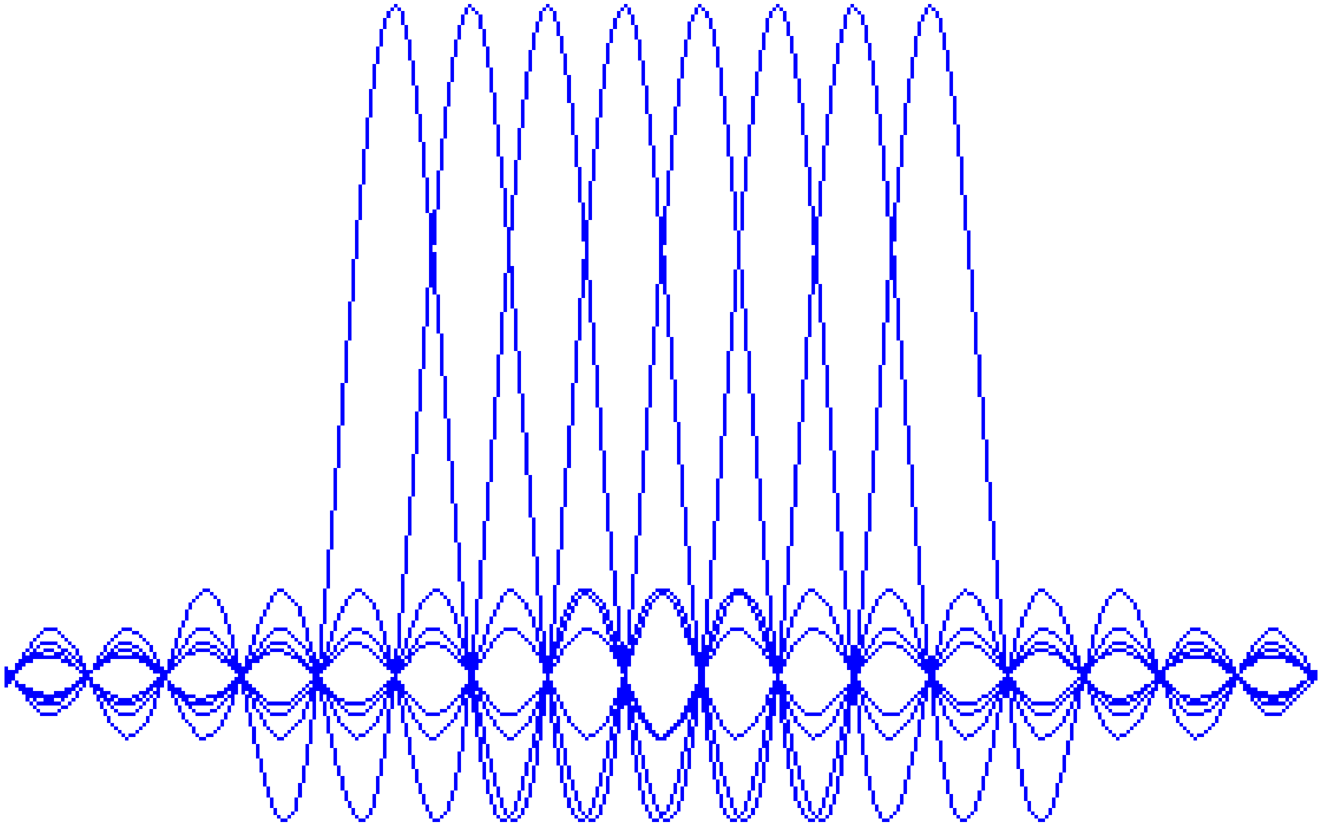
FFT-based OFDM System



OFDM Spectrum



Series and Parallel Concepts

- **In OFDM system design, the series and parallel converter is considered to realize the concept of parallel data transmission.**
- **Series**
  - In a conventional serial data system, the symbols are transmitted sequentially, with the frequency spectrum of each data symbol allowed to occupy the entire available bandwidth.
  - When the data rate sufficient high, several adjacent symbols may be completely distorted over frequency selective fading or multipath delay spread channel.

Series and Parallel concept

- **Parallel**
  - The spectrum of an individual data element normally occupies only a small part of available bandwidth.
  - Because of dividing an entire channel bandwidth into many narrow subbands, the frequency response over each individual subchannel is relatively flat.
  - A parallel data transmission system offers possibilities for alleviating this problem encountered with serial systems.
    - Resistance to frequency selective fading

Modulation / Mapping

- **The process of mapping the information bits onto the signal constellation plays a fundamental role in determining the properties of the modulation.**
- **An OFDM signal consists of a sum of sub-carriers, each of which contains M-ary Phase Shift Keyed (PSK) or Quadrature Amplitude Modulated (QAM) signals.**
- **Modulation type**
  - Phase shift keying
  - Quadrature Amplitude Modulation

Mapping / Phase shift keying

☞ M-ary Phase shift keying

- Consider M-ary phase-shift keying (MPSK) for which the signal set is

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{2\pi(i-1)}{M}\right) \quad 0 \le t \le T_s, \quad i = 1, 2, ..., M$$

where E_s is the signal energy per symbol, T_s is the symbol duration, and f_c is the carrier frequency.

- This phase of the carrier takes on one of the M possible values, namely, $\theta_i = 2(i-1)\pi / M$, where $i = 1, 2, \ldots, M$.

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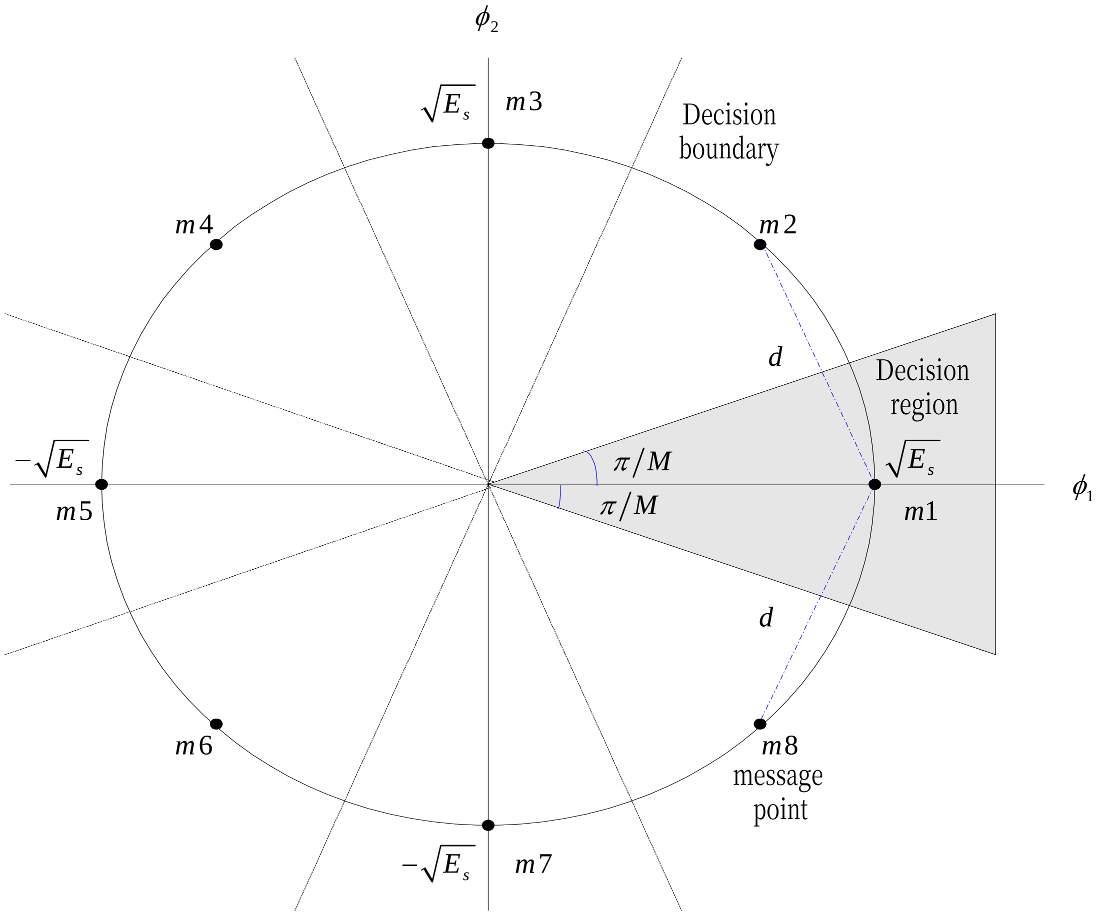
Mapping / Phase shift keying

☞ **An example of Signal-space diagram for 8-PSK .**



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Mapping / Quadrature Amplitude Modulation

- The transmitted M-ary QAM signal for symbol n can be expressed as

$$s_n(t)=\sqrt{\frac{2E}{T}}a_n\cos(2\pi f_c t)-\sqrt{\frac{2E}{T}}b_n\sin(2\pi f_c t),\quad 0\le t\le T,\quad n=0,\pm1,\pm2,....$$

where E is the energy of the signal with the lowest amplitude, a_n and b_n are amplitudes taking on the values

$$a_n,b_n=\pm a,\pm 3a,\ldots,\pm(\log_2 M-1)a \quad ,$$

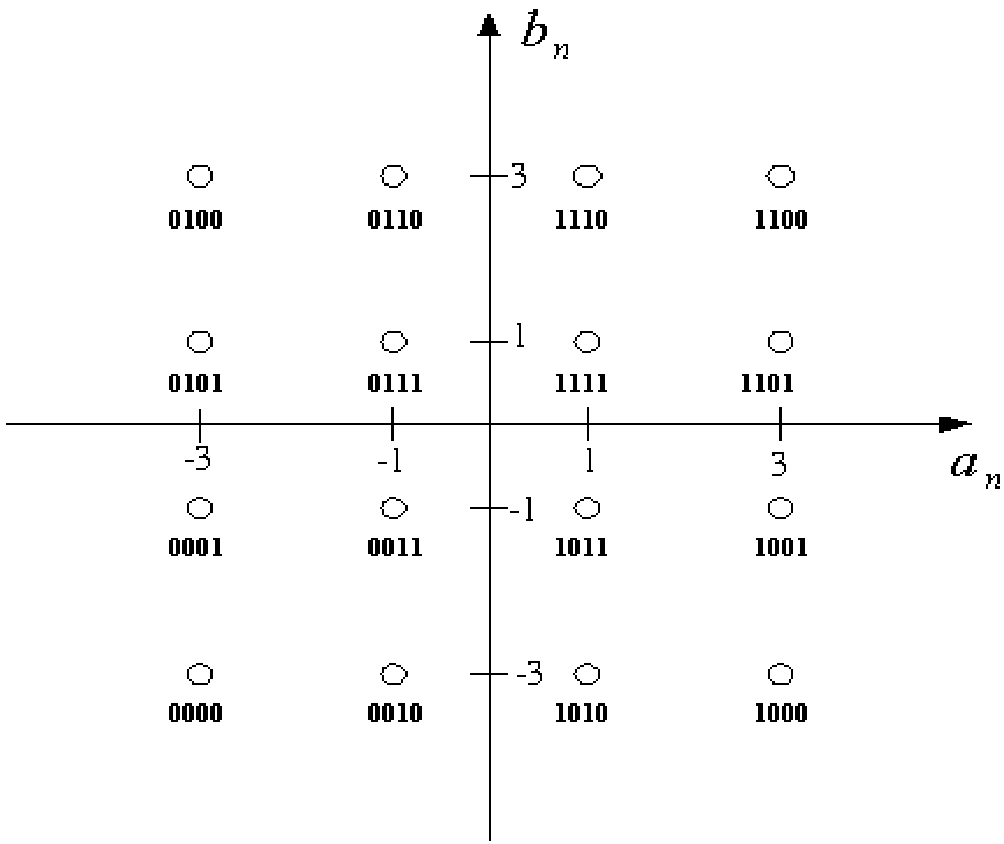
where M is assumed to be a power of *4*.

- The parameter *a* can be related to the average signal energy by

$$a=\sqrt{\frac{3E_s}{2(M-1)}}$$

Mapping / Quadrature Amplitude Modulation

☞ **An example of signal-space diagram for 16-square QAM.**



FFT and IFFT

- **Inverse DFT and DFT are critical in the implementation of an OFDM system.**

$$IDFTx[n] = \frac{1}{N}\sum_{n=0}^{N-1} x[k]e^{-j\frac{2\pi}{N}kn}$$

$$DFTx[k] = \sum_{n=0}^{N-1} x[n]e^{j\frac{2\pi}{N}kn}$$

- **IFFT and FFT algorithms are the fast implementation for the IDFT and DFT.**
- **In the IEEE 802.11a, the size of IFFT and FFT is N=64.**

Signal Representation of OFDM using IDFT/DFT

☞ **Now, consider a data sequence** $d=(d_0,d_1,\cdots,d_n,\cdots,d_{N-2},d_{N-1})$ **and** $d_n = a_n + jb_n$

$$D_m = \sum_{n=0}^{N-1} d_n e^{j(2\pi nm/N)} = \sum_{n=0}^{N-1} d_n e^{(j2\pi f_n t_m)} \qquad m = 0,1,2\cdots N-1 \ ,$$

where $f_n = n/(N\Delta t)$ **,** $t_m = m\Delta t$ **, and** Δt **is an arbitrarily chosen symbol duration of the serial data sequence** d_n **.**

Signal Representation of OFDM using IDFT/DFT

$$s_m = \mathrm{Re}\left(D_m\right)$$

$$= \sum_{n=0}^{N-1}\left(a_n \cos 2\pi f_n t_m - b_n \sin 2\pi f_n t_m\right) \qquad \text{m=0,1,2} \cdots \text{N-1.}$$

- **If these components are applied to a low-pass filter at time intervals**

$$s(t) = \sum_{n=0}^{N-1}\left(a_n \cos 2\pi f_n t - b_n \sin 2\pi f_n t\right) \qquad 0 \le t \le N\Delta t \ .$$

Orthogonality

- **Digital communication systems**
  - In time domain

$$\int_{-\infty}^{\infty} x_i(t) x_j^*(t)\, dt = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

  - In frequency domain

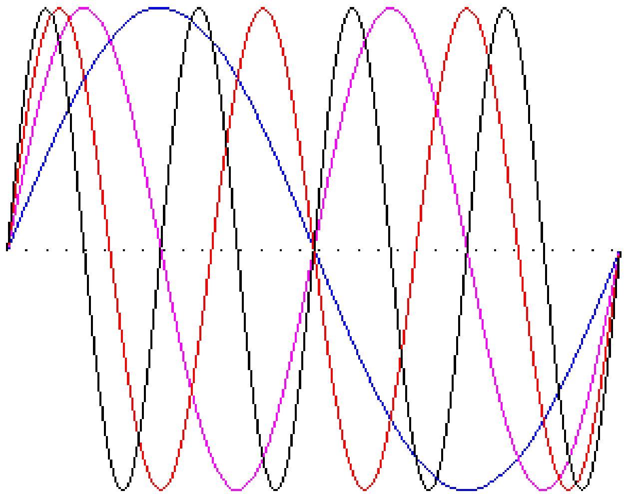
$$\int_{-\infty}^{\infty} X_i(f) X_j^*(f)\, df = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

- **OFDM**
  - Two conditions must be considered for the orthogonality between the subcarriers.
    - 1.Each subcarrier has exactly an integer number of cycles in the FFT interval.
    - 2. The number of cycles between adjacent subcarriers differs by exactly one.

$$\int_{t_s}^{t_s+T} \mathrm{e}^{-j2\pi\frac{k}{T}(t-t_s)} \cdot \sum_{n=0}^{N-1} d_n \mathrm{e}^{j2\pi\frac{n}{T}(t-t_s)} dt = \sum_{n=0}^{N-1} d_n \int_{t_s}^{t_s+T} e^{j2\pi\frac{n-k}{T}(t-t_s)} dt = d_k T$$

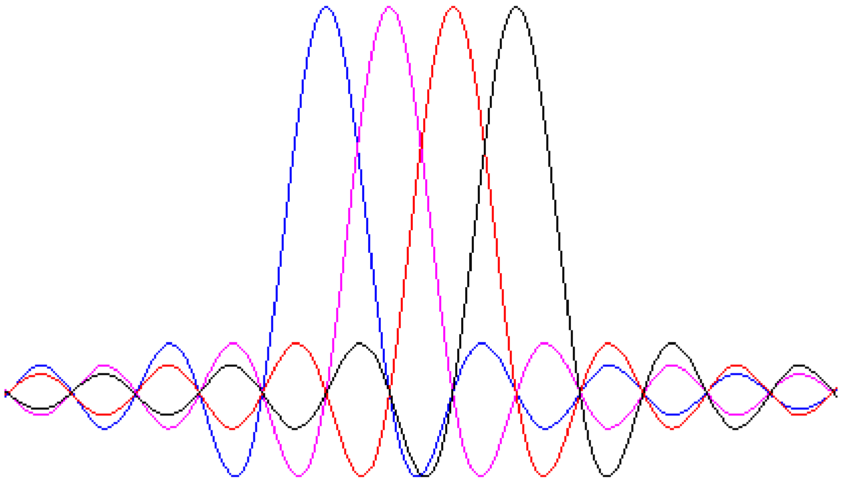
Orthogonality

Time domain



Example of four subcarriers within one OFDM symbol

Frequency domain



Spectra of individual subcarriers

Orthogonality

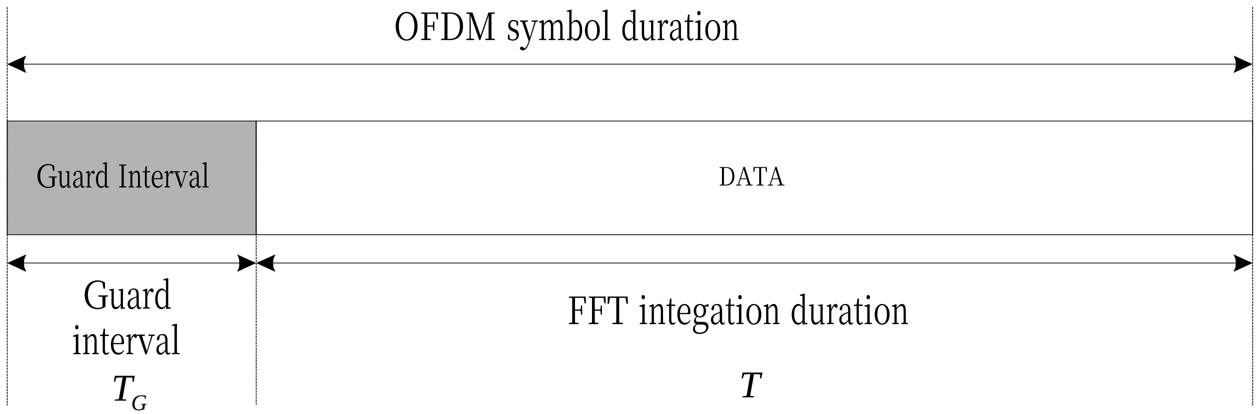
- $\left(e^{j\frac{2\pi}{T_o}t} \quad e^{j2\frac{2\pi}{T_o}t} \quad e^{j3\frac{2\pi}{T_o}t} \quad \cdots e^{jk\frac{2\pi}{T_o}t} \right)$ **are harmonic signals.**
- **Note that harmonics are orthogonal.**
- **The signal** $s(t)$ **is the sum of all harmonic signals.**

$$s(t) = \sum_{t=-\infty}^{\infty} e^{j\frac{2\pi}{T}t}$$

Guard Interval and Cyclic Extension

OFDM Symbol

- OFDM Symbol duration $T_{total} = T + T_g$



Guard Interval and Cyclic Extension

- **Two different sources of interference can be identified in the OFDM system.**
  - **Intersymbol interference (ISI) is defined as the crosstalk between signals within the same sub-channel of consecutive FFT frames, which are separated in time by the signaling interval T.**
  - **Inter-carrier interference (ICI) is the crosstalk between adjacent subchannels or frequency bands of the same FFT frame.**

Guard Interval and Cyclic Extension

☞ **Delay spread**

Environment	Delay Spread
Home	< 50 ns
Office	~ 100 ns
Manufactures	200 ~ 300 ns
Suburban	< 10 us

Guard Interval and Cyclic Extension

- **For the purpose to eliminate the effect of ISI, the guard interval could consist of no signals at all**
- **Guard Interval (or cyclic extension) is used in OFDM systems to combat against multipath fading.**

T_g **:Guard Interval.**

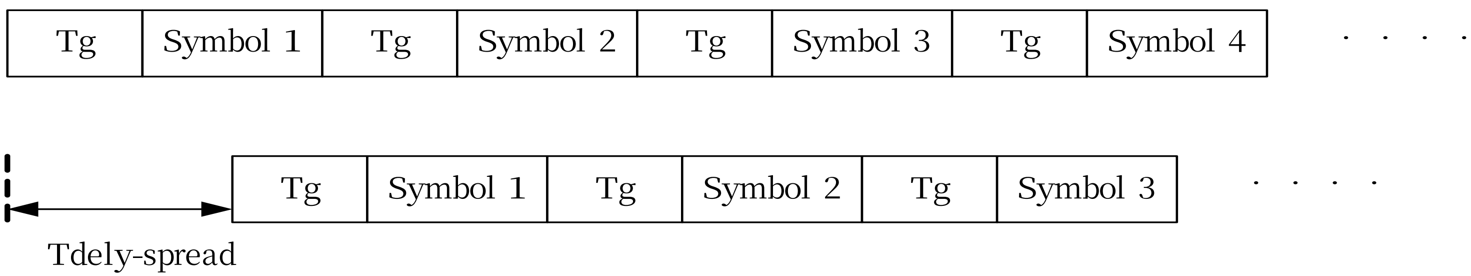
$T_{delay-spread}$ **:Multi Path Delay spread**

$$T_g > T_{delay-spread}$$

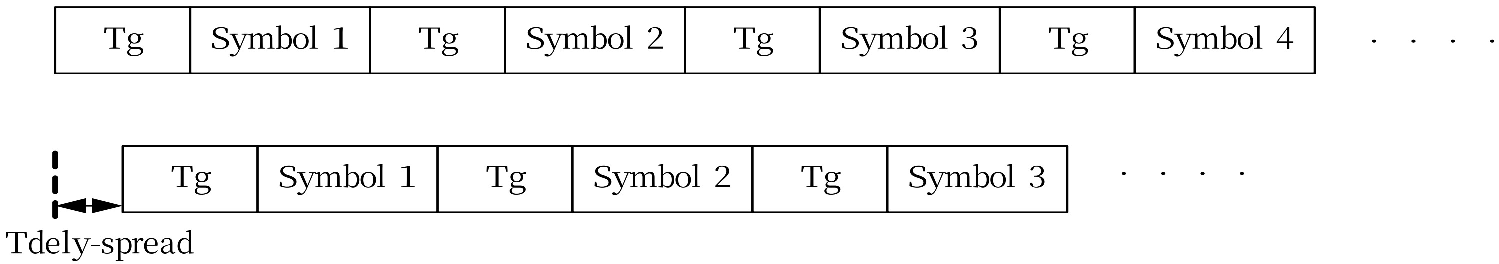
- **In that case, however, the problem of intercarrier interference (ICI) would arise.**
- **The reason is that there is no integer number of cycles difference between subcarriers within the FFT interval.**

Guard Interval and Cyclic Extension

If Tg < Tdely-spread

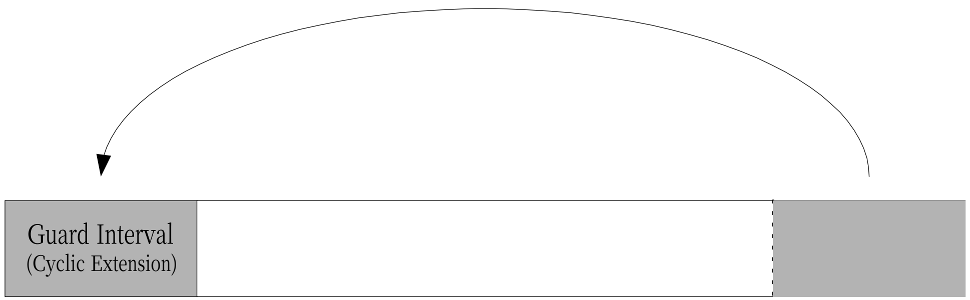


If Tg > Tdely-spread



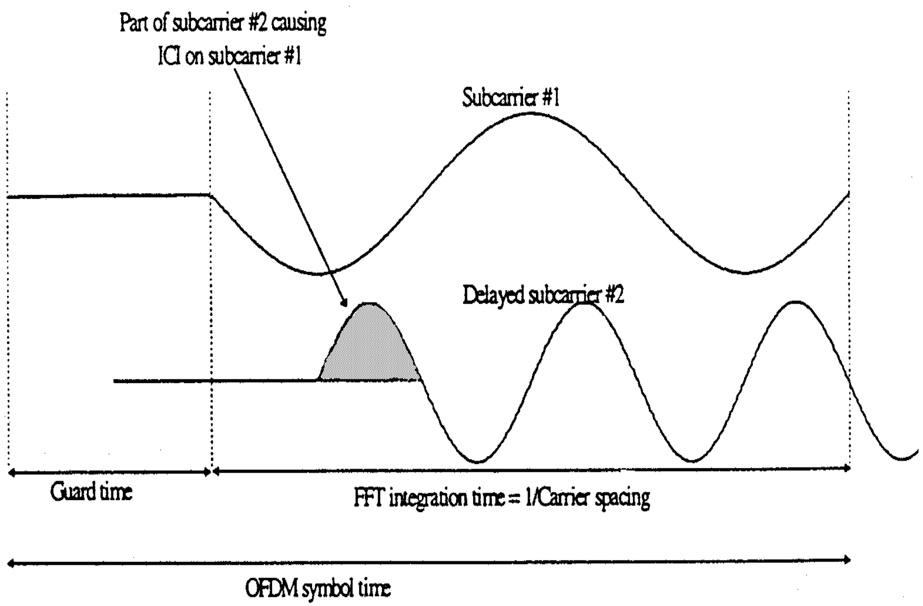
Guard Interval and Cyclic Extension

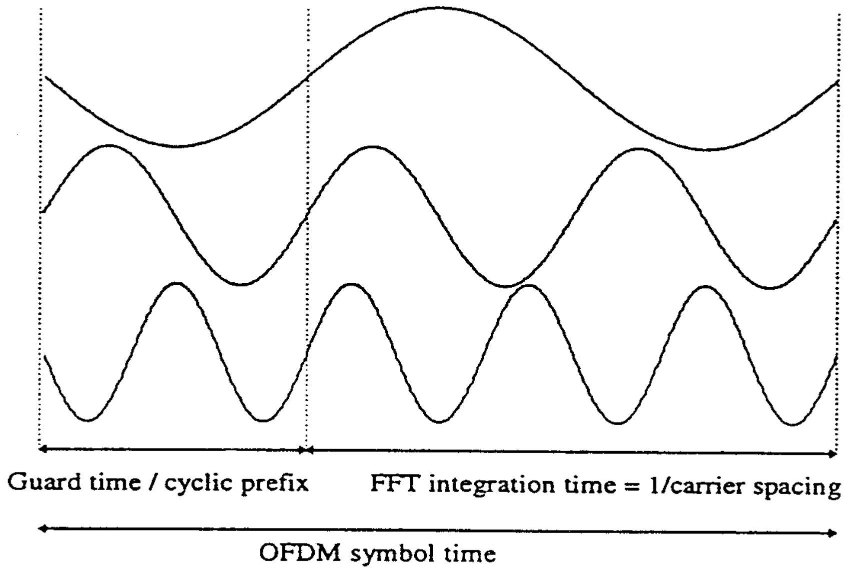
- **The way to eliminate ICI, the OFDM symbol is cyclically extended in the guard interval.**
- **This ensures that delayed replicas of the OFDM symbol always have an integer number of cycles within the FFT interval, as long as the delay is smaller than the guard interval.**



Guard Interval and Cyclic Extension

☞ **Effect of multipath with zero signals in the guard interval, the delayed subcarrier 2 causes ICI on subcarrier 1 and vice versa.**





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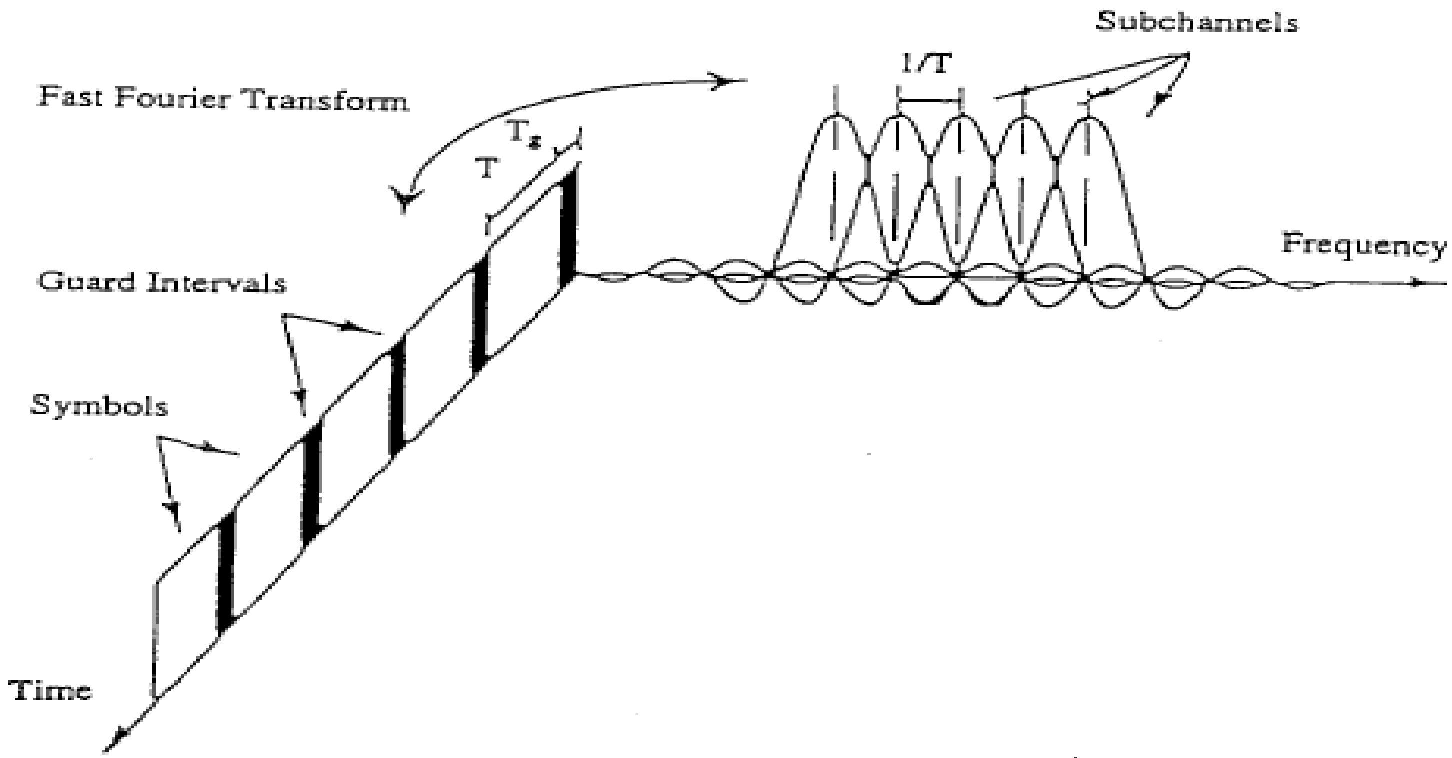
Guard Interval and Cyclic Extension

☞ **Time and Frequency representation of OFDM with guard intervals.**



Advantage

- ☞ **High data rate**
- ☞ **Immunity to Delay Spread**
- ☞ **Resistance to Frequency Selective Fading**
- ☞ **Simple Equalization**
- ☞ **Efficient Bandwidth Usage**

Disadvantage

- ☞ **The Problem of Synchronization**
- ☞ **Need FFT units at transmitter, receiver**
- ☞ **Sensitive to carrier frequency offset**
- ☞ **The problem of High Peak to Average Power Ratio (PAPR)**

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