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# 3A - Impedance Transformation and Impedance Matching

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## References

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- [1] R. Ludwig, P. Bretchko, "RF circuit design - Theory and applications", 2000 Prentice-Hall.
- [2] D.M. Pozar, "Microwave engineering", 2nd edition, 1998 John-Wiley & Sons.
- [3] R.E. Collin, "Foundation for microwave engineering", 2nd edition, 1992, McGraw-Hill.

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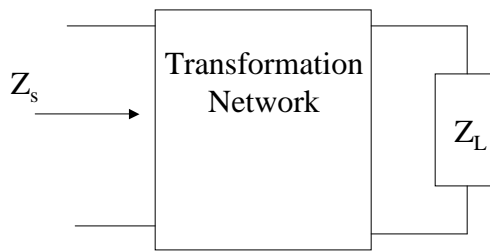
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## Impedance Transformation

- An impedance transformation network is a two-port network that when connected in series with an impedance  $Z_L$  at one port, will result in  $Z_s$  being seen on another port.
- $Z_L$  is usually not equal to  $Z_s$  (otherwise there will be no need for transformation).  $Z_s$  is known as the image impedance of  $Z_L$ .
- We immediately notice that the transformation network is a 2-port network.



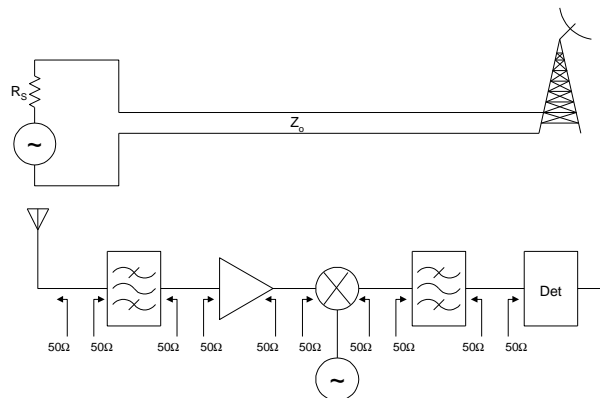
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## Impedance Transformation and Matching



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## Why Impedance Tuning is Needed?

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- Maximum power is delivered when load is matched to the Tline (assuming generator is matched).
- Impedance matching on sensitive receiver components (antenna, low-noise amplifier etc.) improves the signal-to-noise ratio of the system.
- Impedance matching in a power distribution network (such as antenna array feed network) will reduce amplitude and phase errors.

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## Types of Transformation Network

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- Single lumped element (either L or C)
- Dual lumped elements (L impedance matching network)
- Triple lumped elements (Pi or T impedance matching network)
- More lumped elements (ladder type)
- Distributed elements (consists of section of Tlines)
- Hybrid - Consists of both Tline and lumped elements

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## Impedance Transformation Using Lumped Elements

- Lumped components such as surface mounted device (SMD) inductor and capacitor can be easily purchased nowadays.
- SMD capacitors have a range from 0.47pF to greater than 10000pF. With tolerance less than  $\pm 5\%$  and operating temperature between  $-55^{\circ}\text{C}$  to  $125^{\circ}\text{C}$ .
- SMD inductors have a range from 1.0nH to greater than 4000nH. With tolerance from  $\pm 5\%$  to  $\pm 10\%$ , operating temperature from  $-40^{\circ}\text{C}$  to  $125^{\circ}\text{C}$  and Q factor from a minimum of 15 to greater than 45.
- The inductors come in a variety of form, from coil-type, thin-film, to spiral inductors mounted in SMD package. Self-resonance frequency ranges from 200MHz (coil type) for  $L=2200\text{nH}$  to greater than 5GHz for  $L<100\text{nH}$  (thin-film).

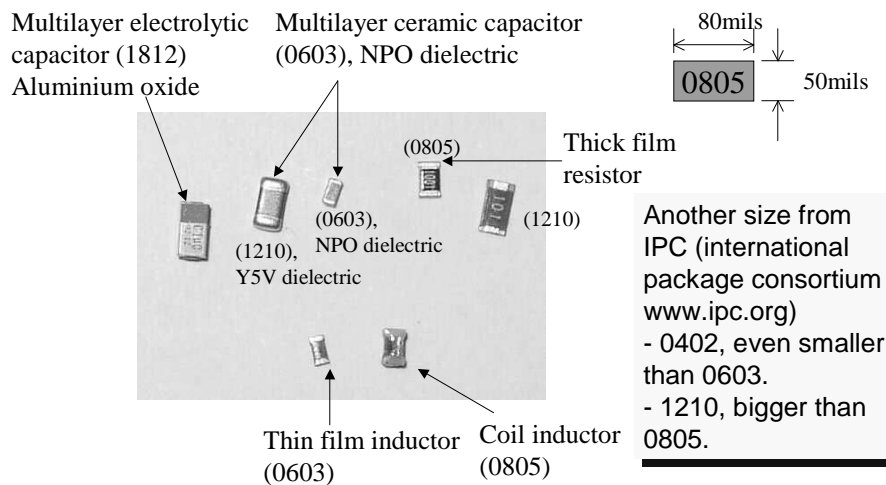
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## Ultra High Frequencies Passive Components (>250MHz)



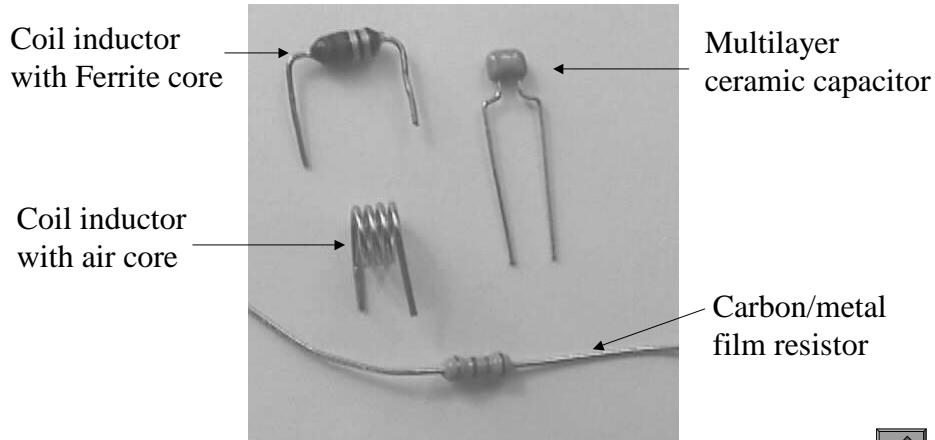
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## Medium Frequency Passive Components (up to 250MHz)



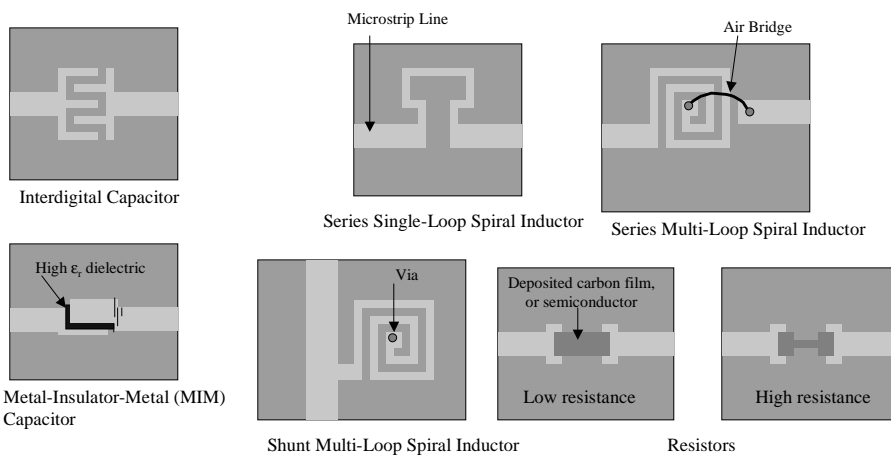
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## Passive Lumped Components for Incorporation into PCB and other Substrates



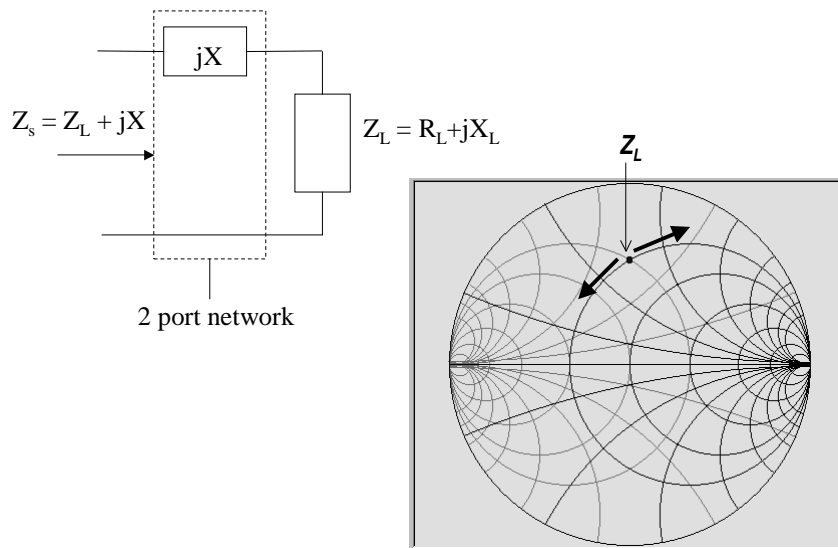
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## Single Lumped Element Transformation Network



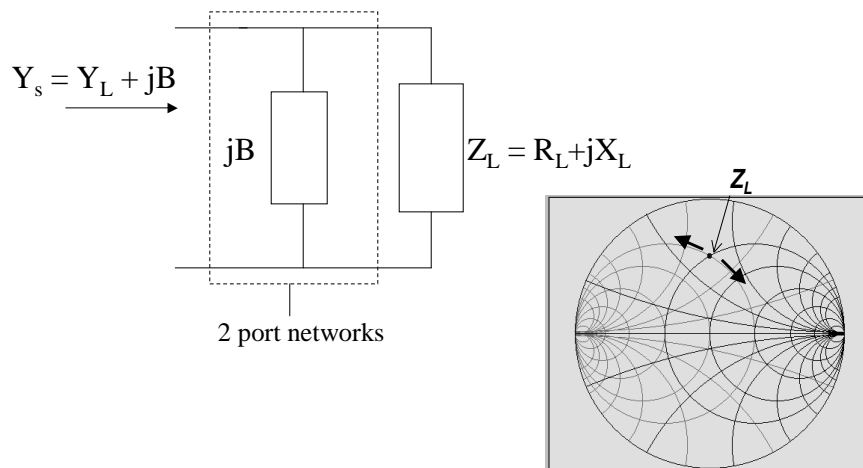
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## Single Lumped Element Transformation Network Cont...



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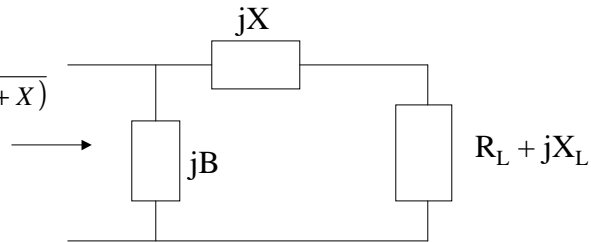
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## Dual Lumped Elements Transformation Network

$$Y_s = \frac{1}{Z_s} = jB + \frac{1}{R_L + j(X_L + X)}$$



If  $Z_s = R_s + jX_s$  is given, we could solve for X and B by equating the real and imaginary parts:

$$X = -X_L \pm \sqrt{R_L(R_s - R_L) + \frac{R_L}{R_s} X_s^2}$$

$$B = \frac{R_s - R_L}{R_s X_L + R_L X_s + R_s X}$$

**(1.1)**

This configuration is only applicable for  $R_s > R_L$

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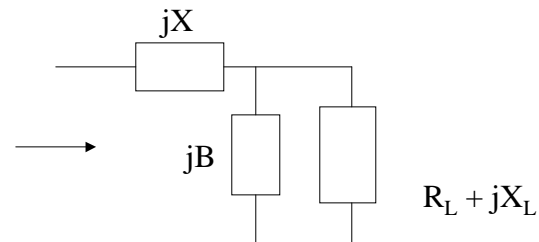
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## Dual Elements Transformation Network Cont...

$$Z_s = jX + \frac{1}{jB + \frac{1}{R_L + jX_L}}$$



If  $Z_s = R_s + jX_s$  is given, again we could solve for X and B by equating the real and imaginary parts:

$$X = X_s \pm \sqrt{R_s(R_L - R_s) + \frac{R_s}{R_L} X_L^2}$$

$$B = \frac{R_s - R_L}{R_L X_s + R_s X_L - R_L X}$$

**(1.2)**

This configuration is only applicable for  $R_L > R_s$

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## Example 1

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- Transform  $Z_L = 100 + j80$  to  $50 + j40$  at 410MHz.

$$X = X_s + \sqrt{R_s(R_L - R_s) + \frac{R_s}{R_L} X_L^2} = 115.498 \quad R_L > R_s$$

$$B = \frac{R_s - R_L}{R_L X_s + R_s X_L - R_L X} = 0.014$$

Since X is +ve, an inductor can be used to realize it:

$$L = \frac{X}{2\pi(410 \times 10^6)} = 44.83nH$$

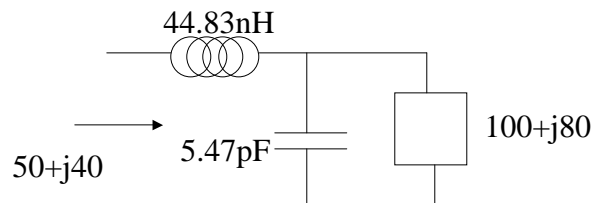
Since B is +ve, a capacitor can be used to realize it:

$$C = \frac{B}{2\pi(410 \times 10^6)} = 5.468pF$$



## Example 1 Cont...

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At 410MHz Only!





## Exercise 1

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- Transform  $Z_L = 50 + j100$  to  $300 - j10$  at 900MHz using 2 lumped element matching networks.

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## Example 2

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- Repeat Example 1 using Smith chart.

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## Exercise 2

- Repeat Example 2 using Smith chart.

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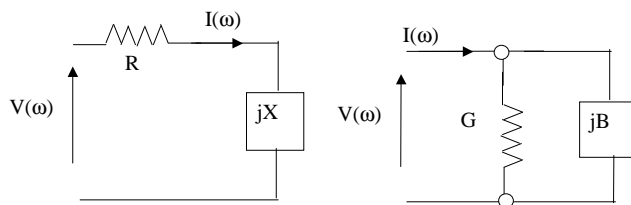


## Q Factor

- The Q Factor of a series or parallel impedance is defined by:

$$Q = \omega \frac{[\text{Maximum energy stored}]}{\text{power dissipation}}$$

*Taken from the notes of EEN2072, Communication Electronics, F. Kung 2000.*



$$Q_s = \frac{|X|}{R} \quad (1.3a)$$

$$Q_p = \frac{|B|}{G} \quad (1.3b)$$

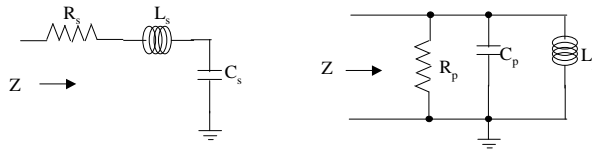
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## Series & Parallel RLC Network



*Taken from the notes of EEN2072, Communication Electronics, F. Kung 2000.*

Parameter	Series RLC network	Parallel RLC network
Input impedance	$R_s + j\omega L_s + \frac{1}{j\omega C_s}$	$\left( \frac{1}{R_p} + \frac{1}{j\omega L_p} + j\omega C_p \right)^{-1}$
Resonance frequency	$\omega_o = \frac{1}{\sqrt{L_s C_s}}$	$\omega_o = \frac{1}{\sqrt{L_p C_p}}$
Quality factor, Q at resonance frequency	$Q_s = \frac{\omega_o L_s}{R_s} = \frac{1}{\omega_o R_s C_s}$	$Q_p = \frac{R_p}{\omega_o L_p} = \omega_o R_p C_p$
Bandwidth BW (note that this is just an approximation)	$\frac{\omega_o}{Q_s}$	$\frac{\omega_o}{Q_p}$

(1.4)

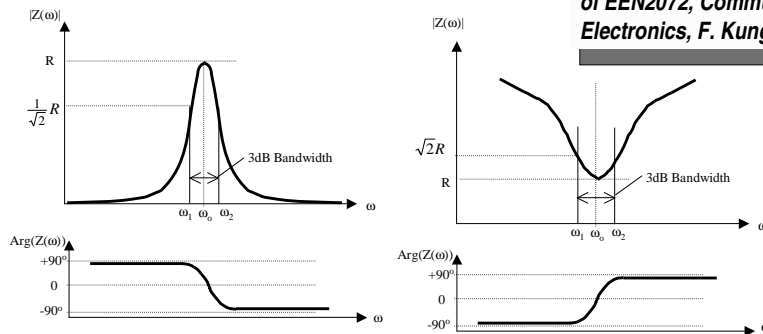
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## Frequency Response of Series & Parallel RLC Network



*Taken from the notes of EEN2072, Communication Electronics, F. Kung 2000.*

**Parallel RLC**

**R ↑ Q ↑**  
**R ↓ Q ↓**

**Series RLC**

**R ↑ Q ↓**  
**R ↓ Q ↑**

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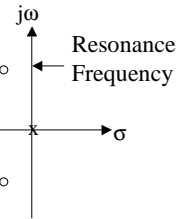
## Poles and Zeros of Series and Parallel RLC Network

**Extra !**

For series RLC:  $Z(\omega) = R + j\omega L + \frac{1}{j\omega C}$

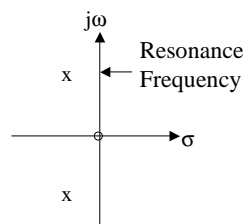
Resonance frequency is the frequency where input impedance to a passive RLC network becomes real.

$= \frac{1 + j\omega RC - \omega^2 LC}{j\omega C}$  ← 2 complex conjugate zeros  
 ← 1 pole on  $j\omega$  axis



For parallel RLC:  $Z(\omega) = \left( \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1}$

1 zero on  $j\omega$  axis →  $\frac{j\omega RL}{R - \omega^2 RLC + j\omega L}$   
 2 complex conjugate poles



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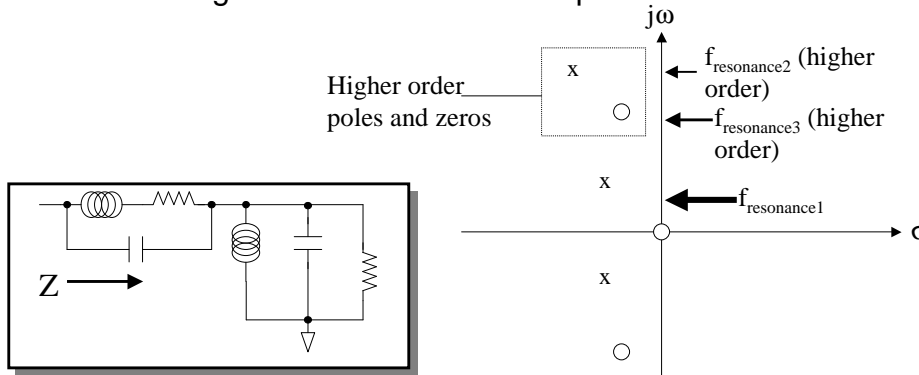
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## Resonance Frequency of Higher Order Systems

**Extra !**

- For a system with more than one L and C, there will be higher order poles and zeros. These will distort the location of the fundamental resonance frequency of the network and introduce higher order resonance frequencies.



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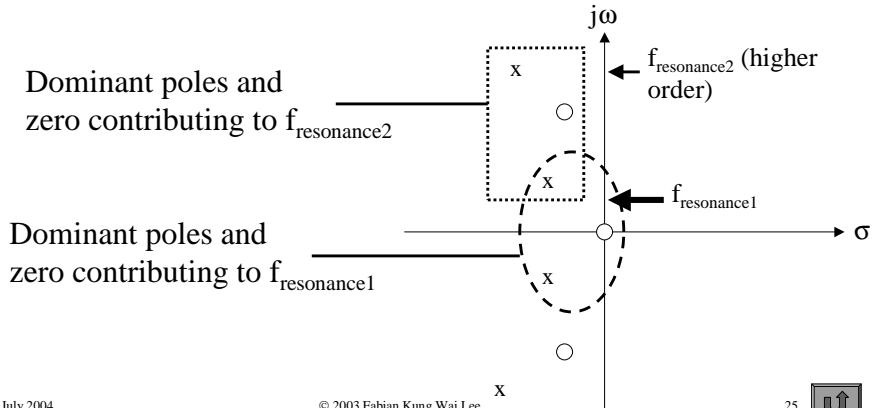
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## Resonance Frequency of Higher Order Systems Cont...

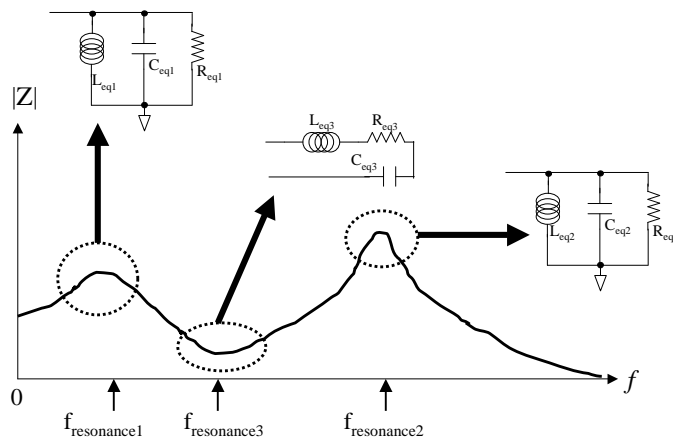
**Extra !**

- Since each resonance frequency is still due to the dominant poles and zeros, the concept of Q factor with regards to 3dB bandwidth can still be applied to higher order network.



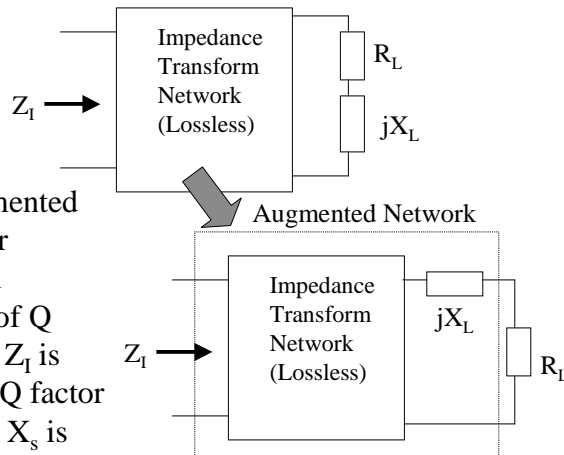
## Resonance Frequency of Higher Order Systems Cont...

**Extra !**



**Extra !**

## Impedance Transformation Network as a Resonating Network



If  $Z_1 = R_s$ , then the augmented network is actually under resonance during normal operation. The concept of  $Q$  factor can be applied. If  $Z_1$  is complex, the concept of  $Q$  factor can still be applied if the  $X_s$  is small.

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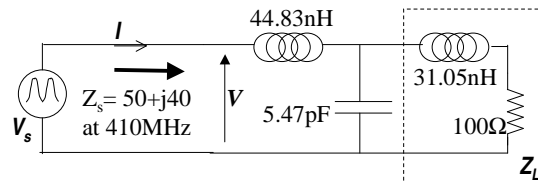
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## Bandwidth of the Matching Network

- Suppose in Example 1 the load  $Z_L$  is actually given by an inductor in series with a resistor, so that at 410MHz we obtain  $Z_L = 100 + j80$ .



- We input the above schematic in a circuit simulator (PSPICE) and run a frequency sweep (change the frequency of the source  $V_s$  while measure  $I$  &  $V$ ) from 100MHz to 800MHz.

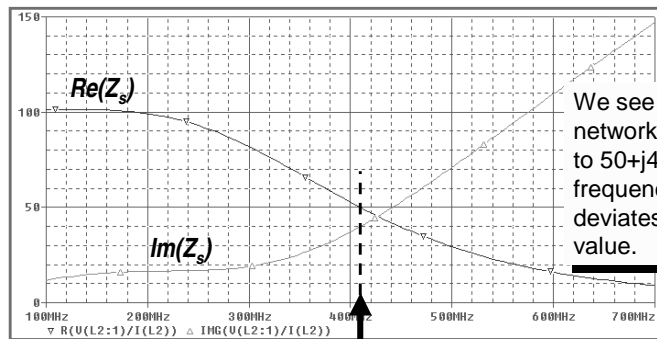
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## Bandwidth of the Matching Network Cont...



410MHz

- Within a range of frequencies near to the operating frequency  $f_o=410\text{MHz}$ ,  $Z_s=R_s + jX_s$  is quite near the desired value. We will call this range of frequency the bandwidth (BW) of the transformation network.

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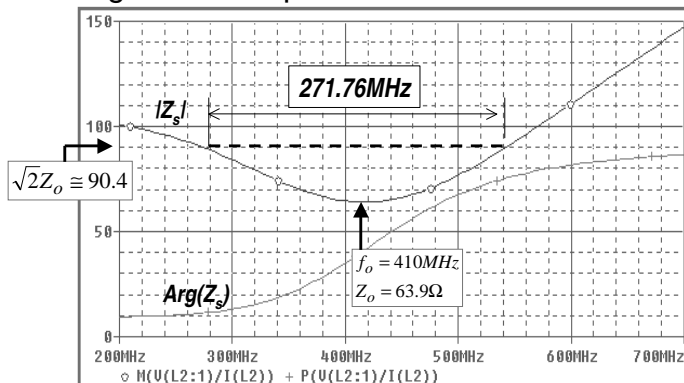
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## Bandwidth of the Matching Network Cont...

- To examine this closer, we plot  $Z_s$  in terms of its magnitude and phase.



$|Z_s|$  and  $\text{Arg}(Z_s)$  is very close to the pattern of series RLC circuit near operating frequency  $f_o$

Following the theory of series RLC network, we define the 3dB BW as the range of freq. Where  $|Z_s|$  is less than  $\sqrt{2}Z_o$ , where  $Z_o$  is the magnitude of the impedance at the operating freq.  $f_o = 410\text{MHz}$ . We see that the 'measured' BW is: BW = 271.76MHz

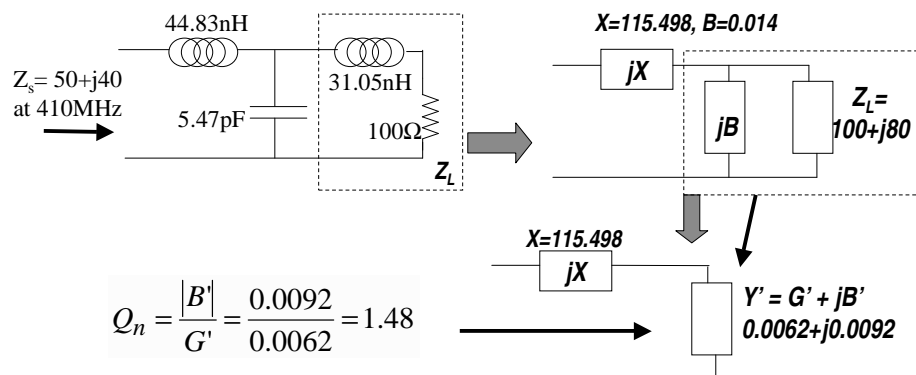
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## Bandwidth of the Matching Network Cont...

- Now consider the circuit of Example 1 again. We could compute a quantity known as the Nodal Q factor,  $Q_n$  as follows:



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## Bandwidth of the Matching Network Cont...

- We could calculate the BW of the system using the equation in (1.4):

$$BW \cong \frac{f_o}{Q_n} = \frac{410 \text{ MHz}}{1.48} = 277 \text{ MHz}$$

**271.76 MHz**

- Surprisingly this is quite near the measured value using simulation. Both measured and calculated BW using this method will match even closer if  $Z_s$  is real, or  $X_s=0$ . This applies to all lumped element transformation network as well (3 elements or more).
- When  $X_s$  is not 0, there is an error, the larger  $|X_s|$ , the greater the error. However this does illustrate that we could in general compare the BW of various transformation network merely by calculating  $Q_n$ .
- High  $Q_n$  denotes narrow BW, low  $Q_n$  denotes wide BW.

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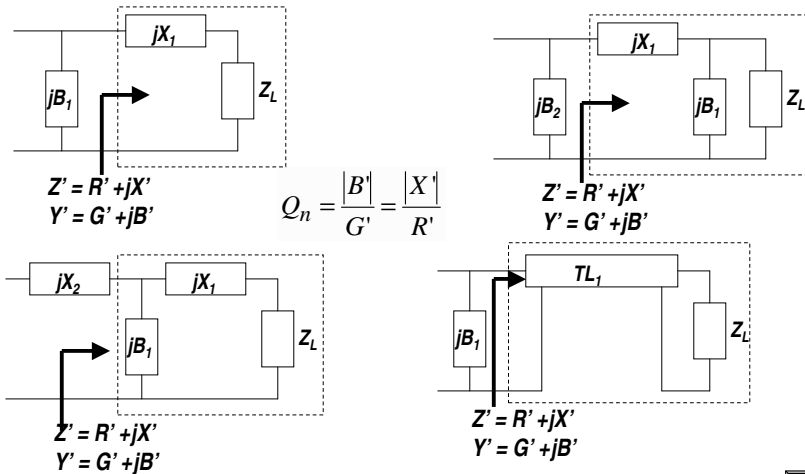
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## Nodal Q Factor, $Q_n$

- $Q_n$  for a few favorite transformation networks.



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## Nodal Q Factor, $Q_n$ Cont...

- The previous slides only illustrate the concept of using nodal Q factor to estimate and compare bandwidth between transformation networks heuristically. A more formal argument and derivation can be found from various materials:
  - R. Ludwig, P. Bretchko, “RF circuit design - Theory and applications”, 2000, Prentice-Hall.
  - J.R. Smith, “Modern communication circuits”, 2nd edition 1998, McGraw-Hill.
  - EEN3096 (Communication Electronics), year 2000 of MMU.
  - Unpublished works of F. Kung, 2003.

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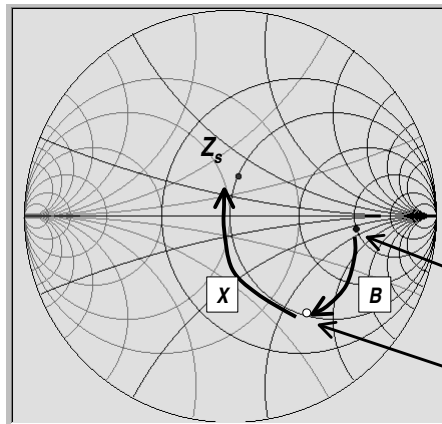
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## Example 3

- Transform the load  $Z_L = 200 - j40$  to  $50 + j20$  at 2.4GHz. Find the nodal Q factor and estimate the bandwidth of the circuit. Use Smith chart to aid the design.



$$Z_s = 50 + j20$$

$$Z_L = 200 - j40$$

$$X = 108.9 \quad B = 0.0008$$

$$L = \frac{108.9}{2\pi(2.4 \times 10^9)} = 7.22 \text{ nH}$$

$$C = \frac{0.008}{2\pi(2.4 \times 10^9)} = 0.53 \text{ pF}$$

$$Q_n = \frac{89.23}{50.58} = 1.764$$

$$BW = \frac{2.4 \text{ GHz}}{1.764} = 1.36 \text{ GHz}$$

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## Constant $Q_n$ Circles

- $Q_n$  depends on the point location on the Smith chart. We could joint all points on the Smith chart giving a similar  $Q_n$  to form a curve or locus. It happens that this locus is a circle, known as Constant  $Q_n$  circles.
- The center and radius for the circles can be derived as follows.
- From section Section 2.2 on Smith chart:

$$r + jx = \frac{1+U + jV}{1-U - jV} = \frac{1-U^2 - V^2}{(1-U)^2 + V^2} + j \frac{2V}{(1-U)^2 + V^2}$$

$$Q_n = \frac{|x|}{r} = \frac{2|V|}{1-U^2 - V^2} \rightarrow U^2 + \left( V \pm \frac{1}{Q_n} \right)^2 = 1 + \frac{1}{Q_n^2}$$

$$\Gamma_{center} = 0 \mp j \frac{1}{Q_n} \quad (1.5)$$

$$Radius = \sqrt{1 + \frac{1}{Q_n^2}}$$

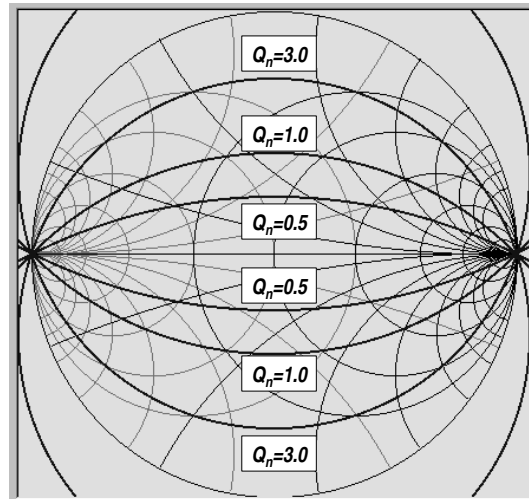
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## Constant $Q_n$ Circles Cont...



$Q_n$	Radius	$1/Q_n$
0.5	2.2360	2.000
1.0	1.4142	1.000
2.0	1.1180	0.500
3.0	1.0541	0.333
5.0	1.0198	0.200

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## Limitation of 2 Lumped Elements Network

- By now it is obvious of the limitation of the 2 elements network. For instance in Example 3 there are only two ways to transform  $Z_L=200 - j40$  to  $Z_s=50 + j20$ .
- Therefore we cannot control the nodal Q factor of 2 elements network, it is determined by the values of  $Z_L$  and  $Z_s$ .
- Using an extra element, we have extra degree of freedom and we can control the value of  $Q_n$  in addition to performing impedance transformation/matching. This is the advantage of using the T or Pi networks.

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## Three or More Lumped Elements Transformation Network

- For more than 3 lumped elements, analytical method such as shown in previous slides is very cumbersome to apply.
- It is more easier to perform 3 elements transformation network design with the aid of Smith Chart.
- As oppose to 2 elements network, 3 or more elements network do not suffer from blind spot. It can transform any passive load  $Z_L$  to any required impedance value.

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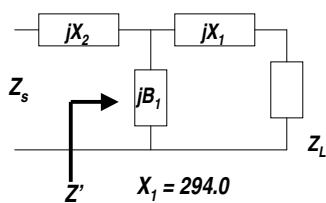
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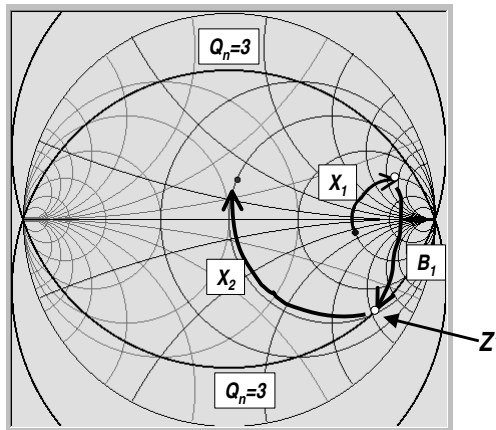
### Example 4

- Repeat Example 3 using 3 elements transformation network, either T or Pi, with the aid of Smith chart. It is required that  $Q_n$  be equal to 3. ( $Z_L=200-j40$ ,  $Z_s=50+j20$ ).



$$\begin{aligned} X_1 &= 294.0 \\ B_1 &= 0.0083 \\ X_2 &= 174.6 \end{aligned}$$

$$\begin{aligned} L_1 &= 19.5\text{nH} \\ C_1 &= 0.55\text{pF} \\ L_2 &= 11.58\text{nH} \end{aligned}$$



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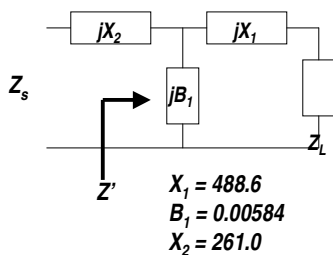
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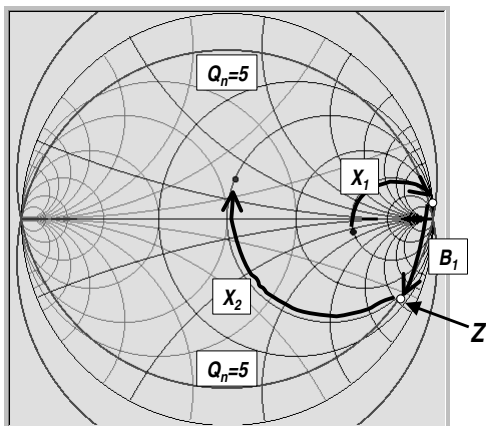


## Example 5

- Repeat Example 4 using 3 elements transformation network, either T or Pi, with the aid of Smith chart. It is required that  $Q_n$  be equal to 5. ( $Z_L=200-j40$ ,  $Z_s=50+j20$ ).



$L_1 = 32.4\text{nH}$   
 $C_1 = 0.387\text{pF}$   
 $L_2 = 17.3\text{nH}$



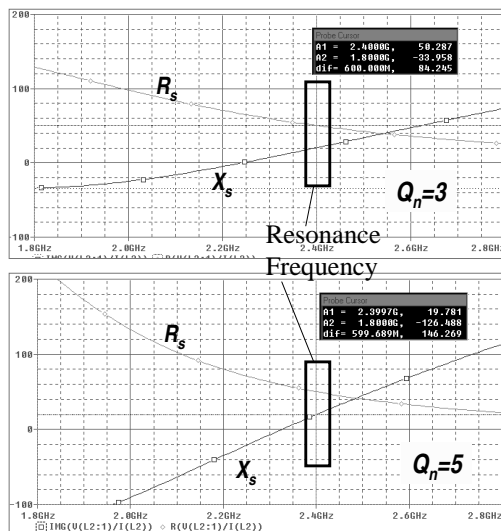
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## $Z_s$ Versus $f$ from Simulation with PSPICE



Both circuits from Example 4 and 5 are fed into PSPICE. AC simulation is run from 1.8GHz to 2.8GHz and the results are compared. It is seen that the T network with higher nodal Q factor has narrower BW, characterized by more rapid deviation from  $f_o = 2.4\text{GHz}$ .

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### Exercise 3

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- Repeat Example 5 using 3 elements T transformation network, with the aid of Smith chart. It is required that  $Q_n$  be equal to 1. ( $Z_L=200-j40$ ,  $Z_s=50+j20$ ). Can you synthesize the T network ? Suggest a solution to this.

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### Exercise 4

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- Repeat Example 5 using 3 elements Pi transformation network, with the aid of Smith chart. It is required that  $Q_n$  be equal to 3. ( $Z_L=200-j40$ ,  $Z_s=50+j20$ ). Can this impedance transformation be realized ? Discuss the result.

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## Pros & Cons of Lumped Element Network

- Lumped element network is compact, small in size.
- Suitable for use up to frequency of 2.5GHz.
- Not every values of inductance and capacitance are available.
- Stability, value changes with temperature.
- Tolerance of components.
- Difficult to tune.
- Higher cost.

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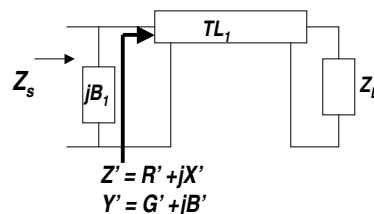
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## Distributed Transformation Network

- Single Stub transformation network.



- $jB$  can be implemented using a Tline with open/short circuit at one end. Can also use lumped elements such as SMD capacitors. In this case the network is known as hybrid network.
- No blind spot.

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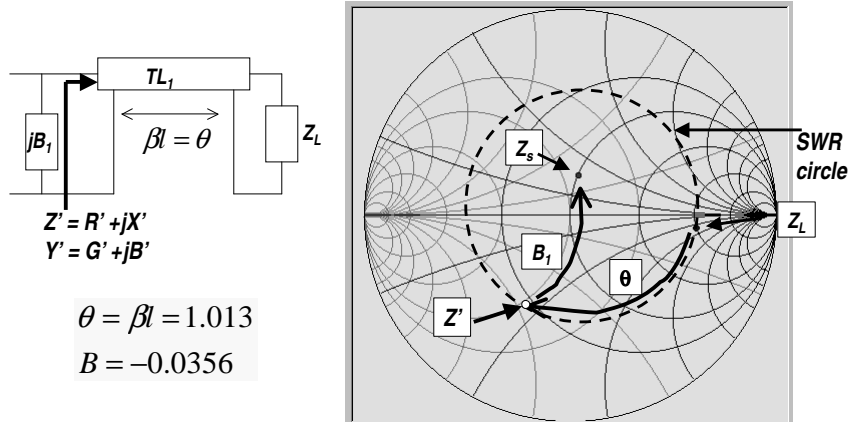
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## Example 6

- Transform the load  $Z_L = 200 - j40$  to  $50 + j20$  at 2.4GHz. Find the nodal Q factor and estimate the bandwidth of the circuit. Use Smith chart to aid the design. Synthesize the circuit.



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## Example 6 Cont...

- Use a microstrip line to implement the circuit,  $Z_c = 50\Omega$ . Dielectric constant = 4.7, and  $d=1.6\text{mm}$ .
- Step 1 - Synthesize Tline.
- From Example 5, Section 3.0, we see that the required  $W$  must be 2.88mm.

$$\begin{aligned}
 \epsilon_{eff} &= 3.55 \\
 \beta &= \omega \sqrt{\epsilon_o \epsilon_{eff} \mu} \\
 &= 2\pi (2.4 \times 10^9) \sqrt{3.55 \epsilon_o \mu_o} = 94.77 \\
 l &= \frac{\theta}{\beta} = \frac{1.013}{94.77} = 0.011 \text{ m} = 1.1 \text{ cm}
 \end{aligned}$$

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## Example 6 Cont...

- Step 2 - Synthesize  $jB_1$ .
- We can use an inductor for  $B_1$ :

$$L = \frac{1}{2\pi(2.4 \times 10^9) \cdot 0.0356} = 1.863 \text{ nH}$$

- Or we can use another short circuit Tline to generate  $B_1$ :

$$Z_{in}(l) = jZ_c \tan(\beta l) = \frac{1}{-jB} = j\left(\frac{1}{B}\right)$$

$$l = \frac{1}{\beta} \tan^{-1}\left(\frac{1}{Z_c B}\right) = \frac{1}{94.77} \tan^{-1}\left(\frac{1}{0.0356 \times 50}\right) = 0.0054 \text{ m}$$

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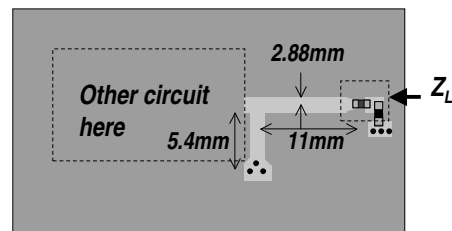
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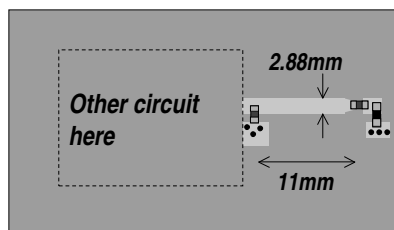


## Example 6 Cont...

- Thus the final circuit...



Or...



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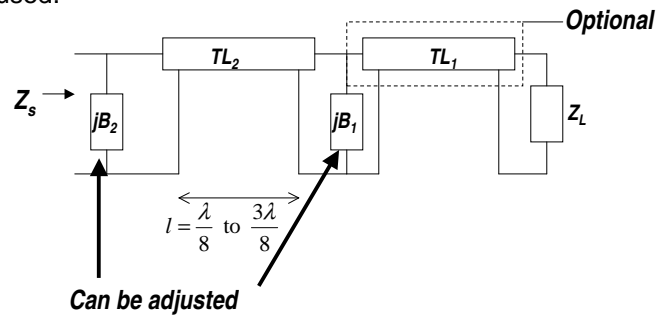
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## Double-Stub Distributed Network

- The single-stub network suffers from the disadvantage of requiring a variable length of Tline between the load and the stub. This may not be a problem for fixed transformation network, but would pose some difficulty if an adjustable tuning network is desired.
- To overcome this disadvantage a double-stub transformation network is used.



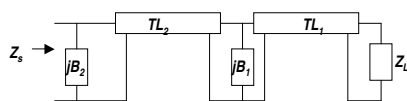
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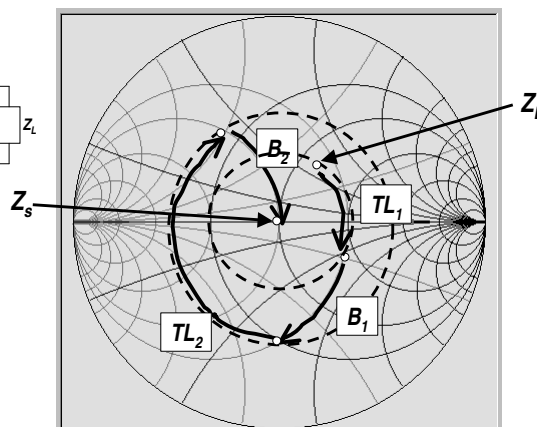
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## Double-Stub Distributed Network Cont...



Suppose we want to transform  $Z_L$  to  $Z_s = 50\Omega$ .



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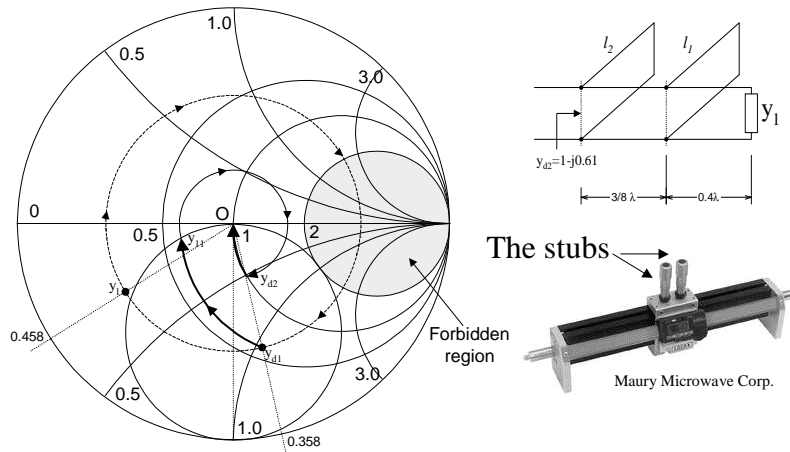
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## Double-stub Matching Cont...

- Double-stub matching using waveguide:



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## Quarter Wave Transformer

- A quarter wave transformer is a simple and useful circuit for matching a real load impedance to a transmission line. An additional feature is that it can be extended to multisection design for broader bandwidth.
- Consider a terminated lossless Tline again, using (1.7) of “2 - Microwave Network Analysis” and letting  $l = \frac{\lambda}{4}$ :

$$l = \frac{\lambda}{4} \Rightarrow \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in}(l) = Z_1 \frac{Z_L + jZ_1 \tan\left(\frac{\pi}{2}\right)}{Z_1 + jZ_L \tan\left(\frac{\pi}{2}\right)} = \frac{Z_1^2}{Z_L}$$

$$\Rightarrow Z_{in}(l) = \frac{Z_1^2}{Z_L} \quad (1.6) \quad \Rightarrow Z_c = \frac{Z_1^2}{Z_L}$$

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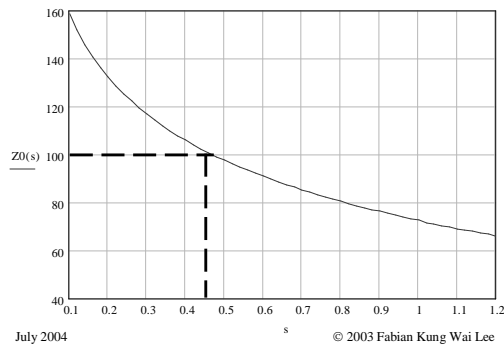
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## Example 7

- Design a quarter wave transformer to transform a  $200\Omega$  load into  $50\Omega$  at  $2.4\text{GHz}$  using a microstrip line constructed on a dielectric with dielectric constant  $4.2$  and thickness  $1.6\text{mm}$ .

$$Z_1 = \sqrt{Z_c R_L} = \sqrt{50 \times 200} = 100$$

Using the microstrip design equations of "1 - Advance Transmission Line":



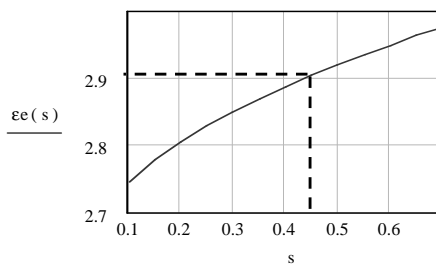
$$\frac{w}{h} = 0.45$$

$$w = 0.45 \times 1.6 = 0.72\text{mm}$$

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## Example 7 Cont...



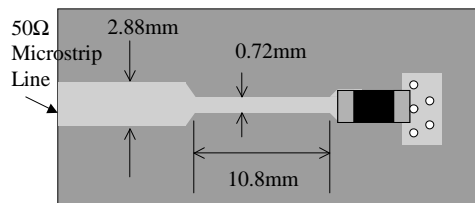
$$\epsilon_e = 2.91$$

$$v_p = \frac{1}{\sqrt{\epsilon_e \epsilon_0 \mu_0}} = 1.75743 \times 10^8$$

$$\beta = \frac{\omega}{v_p} = 85.81$$

$$\beta l = \frac{\pi}{2} \quad \text{For quarter wavelength}$$

$$l = \frac{\pi}{2\beta} = 0.0183 = 10.8\text{mm}$$



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## Quarter Wave Transformer Cont...

- One drawback of the quarter wave transformer is that it can only match a real load impedance, a complex load impedance can always be transformed to a real impedance.
- At the operating frequency  $f_o$ , the electrical length of the matching section is  $\lambda_o/4$ . But at other frequencies the length is different, so a perfect match is no longer obtained. So the quarter wave transformer has a limited bandwidth, like other transformation methods.

- Writing  $Z_{in}$  as:  $t = \tan \theta, \quad \theta = \beta l$

$$Z_{in} = Z_1 \frac{Z_L + jZ_1 t}{Z_1 + jZ_L t} \quad (1.7)$$

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## Extra! BW of Quarter Wave Transformer

- Using (1.6) and (1.7):  $\Gamma = \frac{Z_{in} - Z_c}{Z_{in} + Z_c} = \frac{Z_L - Z_c}{Z_L + Z_c + j2t\sqrt{Z_c Z_L}}$   

$$|\Gamma| = \frac{|Z_L - Z_c|}{\left[ (Z_L + Z_c)^2 + 4t^2 Z_c Z_L \right]^{1/2}}$$

$$= \frac{1}{\left\{ \left[ (Z_L + Z_c)/(Z_L - Z_c) \right]^2 + \left[ 4t^2 Z_c Z_L / (Z_L - Z_c)^2 \right] \right\}^{1/2}}$$

$$= \frac{1}{\left[ 1 + \left[ 4Z_c Z_L / (Z_L - Z_c)^2 \right] \sec^2 \theta \right]^{1/2}} \quad (1.8a)$$

- For frequency near  $f_o$ ,  $l \cong \lambda_o/4$ ,  $\sec^2 \theta \gg 1$ , and this simplifies to:

$$|\Gamma| = \rho \cong \frac{|Z_L - Z_c|}{2\sqrt{Z_c Z_L}} |\cos \theta| \quad (1.8b)$$

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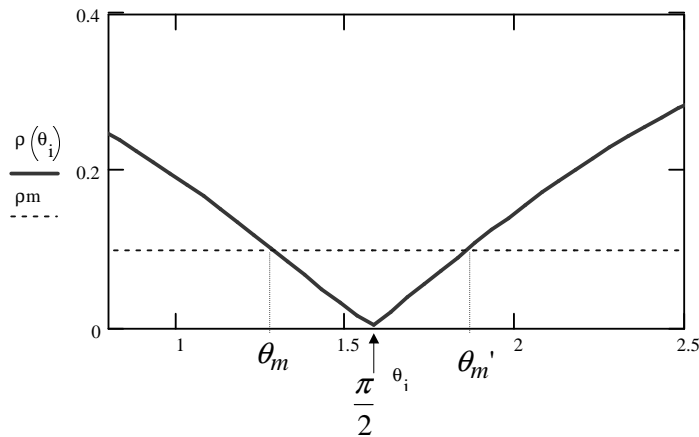
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## BW of Quarter Wave Transformer Cont...

**Extra !**



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## BW of Quarter Wave Transformer Cont...

**Extra !**

- If we set a maximum value,  $\rho_m$ , of the reflection coefficient magnitude that can be tolerated, putting this into (1.8a) and solve for  $\theta_m$  :

$$\cos \theta_m = \frac{\rho_m}{\sqrt{1 - \rho_m^2}} \cdot \frac{2\sqrt{Z_c Z_L}}{|Z_L - Z_c|}$$

- Assuming TEM or quasi-TEM mode:

$$\theta_m = \beta l = \frac{2\pi f_m}{v_p} \cdot \frac{v_p}{4f_o} = \frac{\pi f_m}{2f_o}$$

Quarter wavelength

- And the bandwidth is given by:

$$\begin{aligned} \Delta f &= 2|f_o - f_m| \\ &= 2f_o - \frac{4}{\pi} \cos^{-1} \left[ \frac{\rho_m}{\sqrt{1 - \rho_m^2}} \cdot \frac{2\sqrt{Z_L Z_c}}{|Z_L - Z_c|} \right] \end{aligned} \quad (1.8c)$$

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## Final Note on Quarter Wave Transformer

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- In the previous analysis the reactance associated with the discontinuities must be taken into account.
- Proper compensation technique must be used.

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## Example 8

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- Design a single-section quarter wave transformer to match a 10Ohm load to a 50Ohm Tline, at  $f_0=2.4\text{GHz}$ . Determine the bandwidth for which  $VSWR < 1.3$ . Use the microstrip line of Example 6 to realize it.

$$Z_1 = \sqrt{50 \cdot 10} = 22.361$$

$$\rho_m = \frac{VSWR - 1}{VSWR + 1} = 0.13$$

From example 6



$$\lambda|_{2.4\text{GHz}} = 2\pi / \beta = 6.6\text{cm}$$

$$\beta|_{2.4\text{GHz}} = 94.77$$



$$\frac{\lambda|_{2.4\text{GHz}}}{4} = 1.7\text{cm}$$

$$\left[ 2 - \frac{4}{\pi} \cdot \arccos \left[ \frac{\rho_m}{\sqrt{1 - \rho_m^2}} \cdot \frac{(2 \cdot \sqrt{50 \cdot 10})}{|10 - 50|} \right] \right] \cdot f_0 = 4.511 \times 10^8 \text{ Hz} \Rightarrow \Delta f = 451.2\text{MHz}$$

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## Pros & Cons of Distributed Network

- Easy to fabricate and incorporate into microwave circuit. Utilize the PCB itself.
- Cheap and stable, good tolerance if mechanical tolerance is properly controlled.
- Easier to tune than lumped element network.
- Modern manufacturing facilities use LASER to trim the transmission line dimension during tuning.
- At low frequency, the length of the Tline can be unwieldy large.

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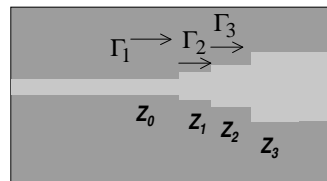
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## Increasing Bandwidth of Distributing Matching Network

- For applications requiring more bandwidth than a single quarter wave section can provide, multi-section transformers can be used.



$Z_n$  must increase or decrease monotonically  
 $Z_L$  must be real.

*The theory of multisection transformer is beyond the time frame of this course. Interested students please refer to Section 5.10-12 of reference [3].*

- We can synthesize any desired reflection coefficient response as a function of frequency, by properly choosing  $\Gamma_n$  and using enough sections.

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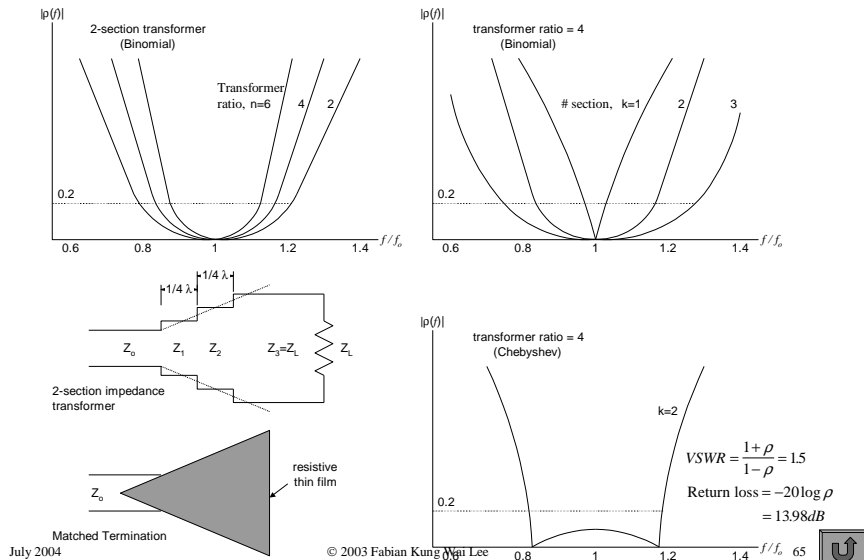
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Extra!

## Multisection Quarter-Wave Transformer



Extra!

## Binomial and Chebyshev Transformers

### Binomial Transformer

- impedance of consecutive  $1/4$  wave lines are proportional to binomial coefficients
- gives maximally flat passband characteristic

### Chebyshev Transformer

- wider bandwidth than Binomial Transformer for the same number of  $1/4$  wave sections
- ripple over passband

### Tapered Transition

- characteristic impedance varies continuously in a smooth fashion
- taper length of  $0.5 - 1.5$  wavelength required

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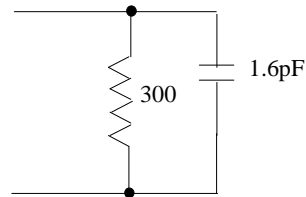
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## Question 1 (16 marks)

- Consider the parallel RC network below. Design a 2-element lumped network that will transform the RC network into  $50\Omega$  at  $900\text{MHz}$ .
- Also determine the nodal Q factor and estimate the operating bandwidth of the transformation network.



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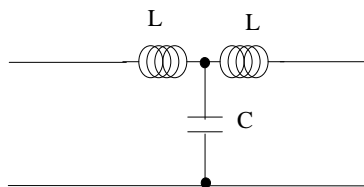
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## Question 2 (14 marks)

- A T network is shown below. Derive, in terms of  $\omega$ , L and C:
- (a) the ABCD matrix of the network.
- (b) the S matrix of the network, take  $Z_{o1}=Z_{o2}=Z_o$  to be  $50\Omega$ .



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