Calculation of PCB Track Impedance

by Andrew J Burkhardt, Christopher S Gregg and J Alan Staniforth

The use of high-speed circuits requires PCB tracks to be designed with controlled (characteristic, odd-mode, or differential) impedances. Wadell[1] is one of the most comprehensive sources of equations for evaluating these impedances. This source includes many configurations including stripline, surface microstrip and their coplanar variants.

The IPC publication, IPC-2141[2], is another source of equations, but has a smaller range of configurations, similar to those presented in IPC-D-317A.

However, for some configurations there are differences between the equations given in these publications. The authors believe that it is now opportune to examine the origin of the equations and to update the method of calculation for use with modern personal computers.

As an example, consider the surface microstrip shown in Figure 1.

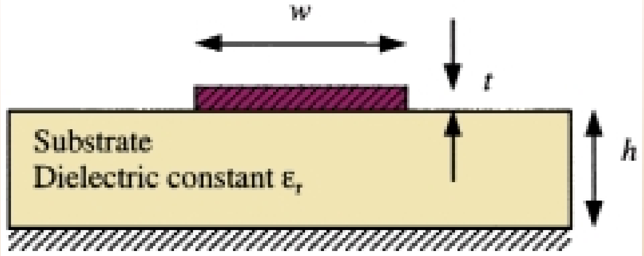
IPC-2141[2] gives the characteristic impedance as

$$Z_0 = \frac{87.0}{(\varepsilon_r + 1.41)^{1/2}} \ln\left[\frac{5.98}{0.8w + t}\right] \quad (1)$$

Wadell[1] gives

$$Z_0 = \frac{\eta_0}{2.0\sqrt{2.0}\pi(\varepsilon_r + 1)^{1/2}} \ln\left[1.0 + \frac{4.0h}{w'}(A + B)\right] \quad (2)$$

Figure 1. Surface Microstrip



where

$$A = \frac{14.0 + 8.0/\varepsilon_r}{11.0} \times \frac{4.0h}{w'} \quad \text{(3a)}$$

$$B = \left(A^2 + \frac{1.0 + 1.0/\varepsilon_r}{2.0} \times \pi^2 \right)^{1/2} \quad \text{(3b)}$$

with

$$w' = w + \Delta w' \quad \text{(3c)}$$

The parameter w' is the equivalent width of a track of zero thickness due to a track of rectangular profile, width w and thickness t. Wadell[1] gives an additional equation to determine the incremental value $\Delta w'$. The parameter η_0, in Equation 2, is the impedance of free-space (or vacuum), 376.7Ω (≈120π). The quoted accuracy is 2% for any value of ´$_r$ and w.

Table 1 shows the results of applying equations (1) and (2) to a popular surface microstrip constructed from 1 oz. copper track on 1/32 inch substrate.

Table 1 shows that Equation 2 is well within the quoted accuracy. The accuracy of Equation 1 varies widely, but this equation has the advantage of simplicity and is useful in illustrating the general changes to the value of Z_0 as the width w and thickness t are varied.

The example demonstrated by Table 1 highlights the general problem with published equations: complicated equations are usually more accurate. Ranges over which the equations are accurate are also usually restricted to a limited range of parameters (e.g., w/h, t/h and ´$_r$).

Equation 2 is complicated, but with patience, can be evaluated using a programmable calculator or computer spreadsheet. However, the complications increase greatly when two coupled tracks are used to give a differential impedance. For coupled surface microstrip, Wadell[1] gives seven pages of equations to evaluate the impedance.

It is now a major exercise to evaluate the impedance using a calculator or spreadsheet.

Figure 3. Stripline: Coplanar Coupled Centered Tracks

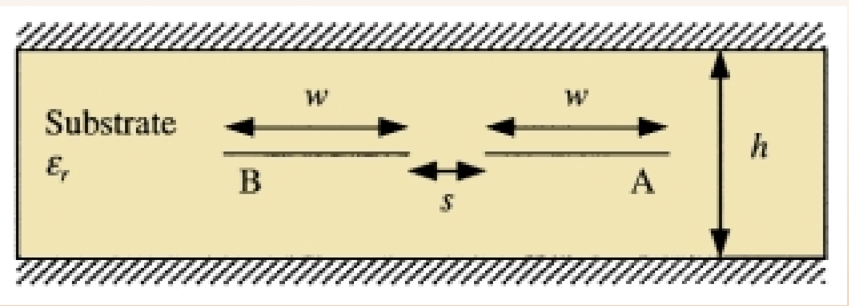
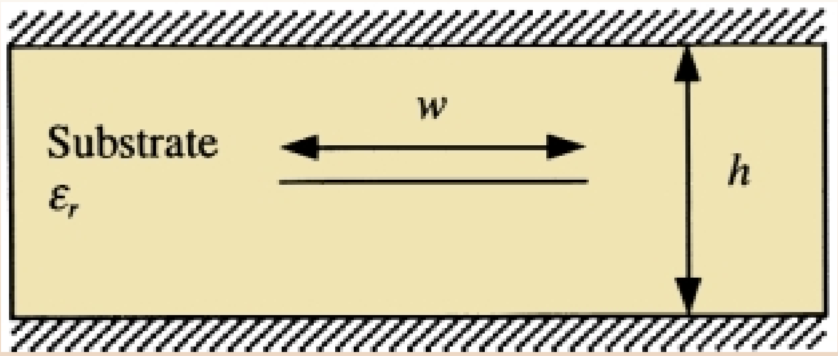


Figure 2. Stripline: Centered Track.



Width w (µm)	Numerical Method Z_0 (Ω)	Equation (1) Z_0 (Ω)	Equation (1) % error	Equation (2) Z_0 (Ω)	Equation (2) % error
3300	30.09	21.08	-29.94	29.89	-0.66
1500	50.63	49.46	-2.31	50.50	-0.26
450	89.63	91.79	+2.41	89.89	+0.29

t = 35µm, h = 794µm, ε_r = 4.2
(the calculation of the error assumes the numerical method is accurate : see Numerical Results)
Table 1

ALGEBRAIC EQUATIONS

Single Track

For the stripline of Figure 2 with a symmetrically centered track of zero thickness, Cohn[3] has shown that the exact value of the characteristic impedance is

$$Z_0 = \frac{\eta_0}{4.0\sqrt{E_r}} \frac{K(k)}{K(k')} \quad \text{(4)}$$

where

$$k = \operatorname{sech}\left(\frac{\pi w}{2.0h}\right) \quad \text{(5a)}$$

and

$$k' = \tanh\left(\frac{\pi w}{2.0h}\right) \quad \text{(5b)}$$

K is the complete elliptic function of the first kind[4]. An equation for the evaluation of the ratio of the elliptic functions, accurate to 10^{-12}, has been given by Hilberg[5], and also quoted by Wadell[1].

When the thickness is not zero, corrections have to be made which are approximate[1]. These corrections are obtained from theoretical approximations or curve fitting, the results of numerical calculations based on the fundamental electromagnetic field equations.

When the track is offset from the center, the published equations become more complicated and the range of validity, for a given accuracy, is reduced.

Attempts have also been made to include the effects of differential etching on the track, resulting in a track cross-section which is trapezoidal[1].

There is no closed-form equation like Equation 4 for surface or embedded microstrip of any track thickness. Thus, any equation used to calculate the impedance is approximate and demonstrated in Table 1.

Coupled Coplanar Tracks

Figure 3 shows two coupled coplanar centered stripline tracks.

Coupled Centered Tracks

All the impedance equations for coupled configurations refer to both even-mode impedance (Z_{0e}) and odd-mode impedance (Z_{0o}). These impedances are measured between the tracks and the ground plane. Z_{0e} occurs when tracks A and B are both at +V relative to the ground plane, and Z_{0o} occurs when track A is at +V and track B is at –V. When a differential signal is applied between A and B, then a voltage exists between the tracks similar to the odd-mode configuration. The impedance presented to this signal is then the differential impedance,

$$Z_{diff} = 2 \times Z_{0o} \qquad (6)$$

All published equations [1] give Z_{0o}. The differential impedance must then be obtained using equation (6).

For the zero thickness configuration of Figure 3, Cohn[3] gives the exact expression.

$$Z_{0o} = \frac{\eta_0}{4.0\sqrt{\varepsilon_r}} \frac{K(k_0)}{K(k'_0)} \qquad (7)$$

where

$$k_0 = \left(1 - k'^2_0\right)^{1/2} \qquad (8a)$$

and

$$k'_0 = \tanh\left[\frac{\pi w}{2.0h}\right] \coth\left[\frac{\pi(w+s)}{2.0h}\right] \qquad (8b)$$

As before, K is the elliptic function of the first kind. There are no closed-form equations for coplanar coupled tracks.

Effect of Track Thickness

When the track thickness is not zero, approximations must be made to obtain algebraic equations similar to Equations 4 and 7. Alternatively, equations based on curve fitting of extensive numerical calculations are used.

However, as the thickness increases the impedance decrease, as can be noted from Equation 1.

Numerical Principles

For pulses on a uniform transmission system [1,6] then

$$Z_0 \quad (or\ Z_{0o}) \quad = \sqrt{\frac{L}{C}} \qquad (9)$$

where L is the inductance and C the capacitance per unit length of line.

For a stripline, where the electric (and magnetic) fields are in a uniform substrate, dielectric constant ´r, Equation 9 becomes

$$Z_0 = \frac{\sqrt{\varepsilon_r}}{cC} \qquad (10)$$

where c is the velocity of light in vacuuo (or free-space). The velocity of pulse travel along the transmission path is

$$v = \frac{c}{\sqrt{\varepsilon_r}} \qquad (11)$$

FoFor a microstrip, the electric (and magnetic) fields are in air and the substrate. It can be shown that

$$Z_0 = \frac{1}{c\sqrt{CC_{air}}} \qquad (12)$$

Where C_{air} is the capacitance of the same track configuration without substrate. The effective dielectric constant is

$$\varepsilon_{eff} = \frac{C}{C_{air}} \qquad (13)$$

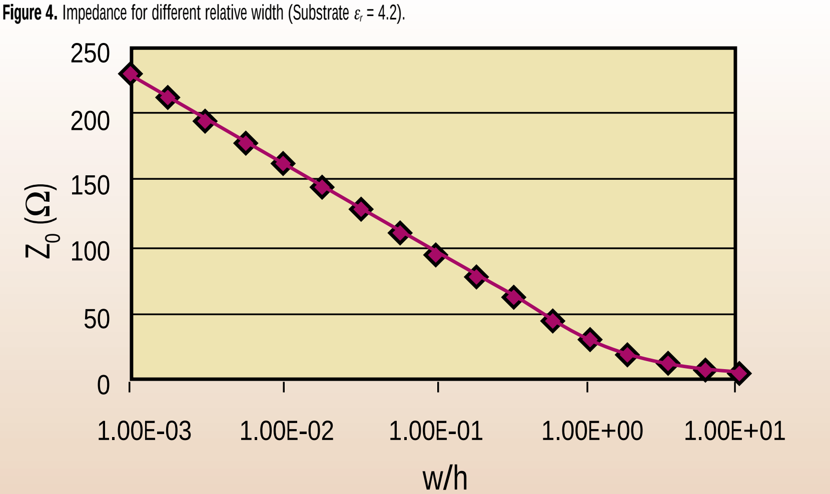
To find the impedance, the capacitance must be calculated. This can be done by applying a voltage V to the tracks and calculating the total charge per unit length Q, from which

$$C = \frac{Q}{V} \qquad (14)$$

However, the surface charge on a track is not uniform. In fact, it is very high at track corners. Therefore, the total charge is difficult to calculate.

From electrostatic theory, it is known that a charge produces a voltage at a distance r from the charge. Then a distribution of charge r (coulomb/unit width of track) gives a voltage

Figure 4. Impedance for different relative width (Substrate ε_r = 4.2).



$$V = \int G \rho \delta l \quad (15)$$

where the integral is taken over the perimeter of the track cross-section, d*l* is a small length, and *G* is the voltage due to a unit charge. It is also known as the Green's Function. The value of *G* depends on the configuration (or environment). For instance, a point charge in a two-dimensional dielectric space, without conductors gives

$$V = -\frac{\rho \ln(r)}{2\pi\varepsilon_0\varepsilon_r} \quad (16a)$$

so that

$$G = -\frac{\ln(r)}{2\pi\varepsilon_0\varepsilon_r} \quad (16b)$$

In equation (15), the voltage *V* is known, *G* is known for the particular configuration of tracks and substrate, but the charge r is unknown. Thus (15) is an integral equation which can be solved numerically by the Method of Moments (MoM)[7].

To proceed using MoM, the cross-section perimeter of the track is divided into short lengths with a node at each end. Charges are assigned to each node. The voltage at each node is calculated from all the nodal charges and the estimated charge variation between nodes. This leads to a set of simultaneous equations represented by the matrix equation

$$\mathbf{A}\,\rho = \mathbf{V} \quad (17)$$

where r is a vector of nodal charges, and *V* is a vector of nodal voltages. *A* is a square matrix whose elements are calculated from integrals involving the Green's Function. The size of the matrices depends on the number of nodes. Equation 17 can be solved for the nodal charges r for given nodal voltages *V*. The elements of *V* are usually +1 or –1 depending on the configuration.

The total charge *Q* can be obtained by a suitable summation of the nodal charges.

This general approach has been used by most authors to evaluate the various impedances. Most of the calculations were published fifteen to twenty years ago, when the principal calculator was a mainframe computer—hence the need for equations which could be used with the pocket calculators available at that time.

The present authors have revisited the basic numerical approach and have developed software[8] which readily calculates the controlled impedances using a desktop PC. The software runs quickly on a modern PC, and has been extended to also include the calculation of configurations not well represented in the literature. This includes

- offset coupled stripline
- broadside coupled stripline
- embedded coupled microstrip

Thick tracks are normally to be expected which have a trapezoidal cross-section to allow for differential etching of the track.

Figure 5. Substrate ε_r = 4.2

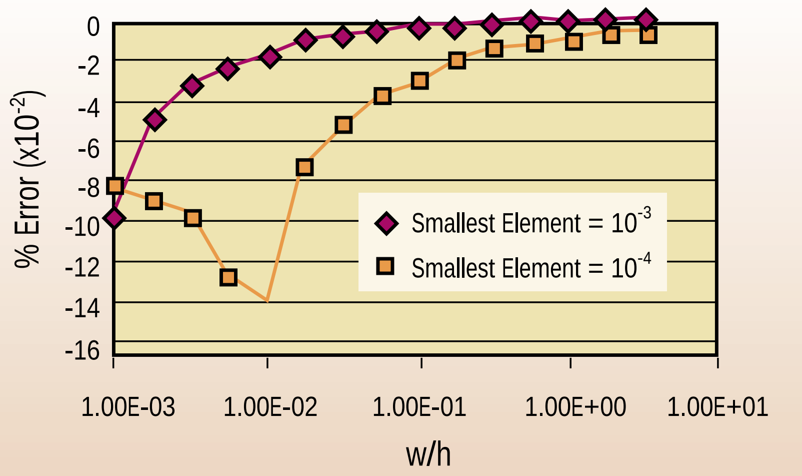
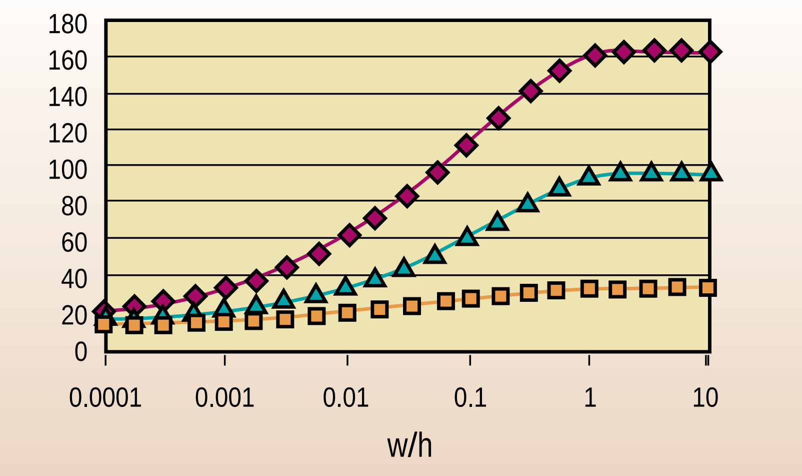


Figure 6. Odd-mode impedance for different separations (s/h) and widths (w/h)



Numerical Results

This section describes in more detail some of the numerical techniques and compares the results with the exact Equations 4 and 7.

In all cases, the Green's Function for the configurations was obtained using charge images in the ground planes. There are an infinite number of these images. In the case of stripline the sum of images converges to the result given by Sadiku[9]. Silvester[10, 11] developed the image method for surface microstrip and has now been extended by the authors for embedded microstrip. In all cases, the sum of images converges, but the result has to be obtained numerically.

The distribution of charge over an element between nodes is assumed to be linear. A numerical singularity occurs when the charge node *j* coincides with the voltage node *i*. Sadiku[9] indicated how this can be resolved. The evaluation of the elements A_{ij} consists of both numerical and analytic integration in the same manner as that used in Boundary Element techniques[12, 13].

To avoid numerical inaccuracies at corners where there is a large concentration of charge, the length of an element at a corner is made very small. The other elements and nodes are then distributed by the method described by Kobayashi[14]. This means that wide strips require more nodes than narrow strips when the same small element is used.

The results presented were performed on a PC with an Intel Pentium Pro running at 233MHz using a compiled C-program.

Single Track Stripline

Figure 4 shows the variation of impedance with track width for the stripline of Figure 2.

Figure 5 shows the error percentage of the numerical calculation compared with the exact values given by Equation 4. Two curves are shown for different small elements at the corner (i.e. ends of the track).

The above graph shows that good accuracy can be obtained over nearly four decades of the width/height ratio. The computer processing time was less than 0.5s for any of these values.

Coupled Coplanar Stripline

Figure 6 shows the variation of the odd-mode impedance for the stripline shown in Figure 3.

Figure 7 shows the error percentage of the numerical calculation compared with the exact values given by equation (7) using 10^{-3} as the smallest element. The maximum processing time was less then 0.5s. The maximum error can be reduced by decreasing the smallest element. For a maximum error of $6.0x10^{-2}$%, a processing time of 5.1s is required.

The results presented in Figure 7 offer a very stringent test for the numerical method because of the sharp corners separated by s. In the odd-mode configuration, this effect is enhanced even more because the tracks are of opposite polarity. This numerical validation is considered to be better then the results given by Bogatin et. al.[15] for a pair of 'round' tracks (i.e. a parallel wire transmission line) using finite ele-

After communicating with the authors of this paper, I decided to add a second table (Table 1, below) that represents dimensions closer to those of HDI substrates. Again, the table shows the results of applying equations (1) and (2) to the surface microstrip constructed from 1 oz. copper on thinner HDI dielectrics. Notice again that equation (1) varies from the numerical method, and again equation (2) has very good accuracy. It is advisable not to use the IPC equations when working with HDI structures.

Equation (2) is quite accurate for simple microstrips but similar accurate equations do not exist for more complex HDI structures, like differential impedance or coplanar structures. Figure 1 shows 16 such high-speed signal models that can best be solved by numerical methods such as 2-D or 3-D field solvers.

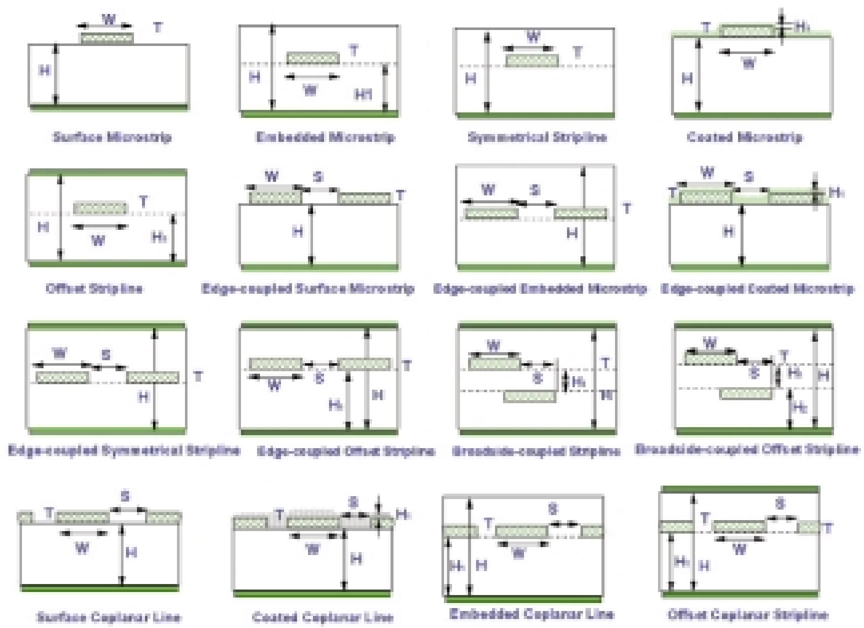


Figure 1. Examples of 16 high-speed impedance models that require numerical methods to calculate impedance.

Width w (μm)	Numerical Method Zo (Ω)	Equation (1)		Equation (2)	
		Zo (Ω)	% error	Zo (Ω)	% error
150	69.41	61.00	-12.12	70.10	0.99
100	52.13	45.30	-13.10	52.32	0.36
50	42.09	34.33	-18.44	42.06	-0.07

t = 35μm, h = 66μm, er = 4.2
(the calculation of the error assumes the numerical method is accurate : see Numerical Results)

Table 1. Comparison of various methods of determining microstrip impedance.

ment software. In this latter case, there are no singularities at the corners. Li and Fujii[16] state that the boundary element method (to which MoM is related) is more accurate for stripline and microstrip than the finite element method.

Surface Microstrip

As previously mentioned, there are no closed-form algebraic equations which are exact. But the discussion in the previous sections shows that the software can be made accurate, especially for practical purposes. Table 1 shows calculations for the configuration of Figure 1. Because the Green's Function involves a summation, and two capacitances C and C_{air} are required, processing times are now longer than those for stripline. The longest time was less than 4.5s for a width of 3300µm.

For coupled surface microstrip, two thick tracks of 3300µm requires a processing time of 5.1s. The separation does not affect the time.

Practical Results

In order to verify the practical performance of the field solving boundary element method, the authors commissioned production of a set of samples. During a six-month period in 1998, over 1500 different printed circuit board tracks were manufactured.

This sample consisted of both stripline and microstrip differential structures in surface and embedded configurations. Two types of coupled structures were included; edge-coupled and boardside-coupled. The track dimensions ranged from 75µm to 1000µm in width, with differential separations of one track width to four track widths using base copper weights of 1/2 oz., 1 oz. and 2oz. The resulting differential impedances ranged from 80Ω to 200Ω.

Test samples were produced by three independent UK printed circuit board manufacturers[17] and the differential impedances were electrically measured by TDR at Polar Instruments using a CITS500s Controlled Impedance Test System.

After electrical measurement, the samples were returned to the manufacturers for microsection analysis to determine the actual physical mechanical dimensions.

The calculated impedance was predicted from the mechanical microsection data and a derived value of relative permitivity, ε_r, of the FR-4 material. Results[18] were analyzed,

Figure 7. % error ε_r = 4.2).

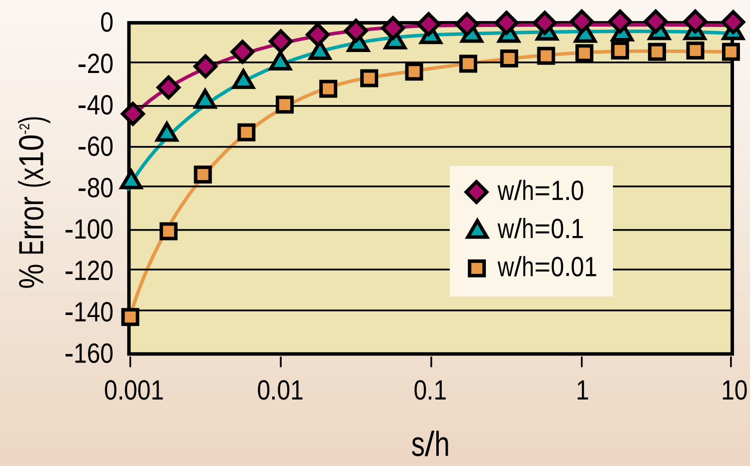
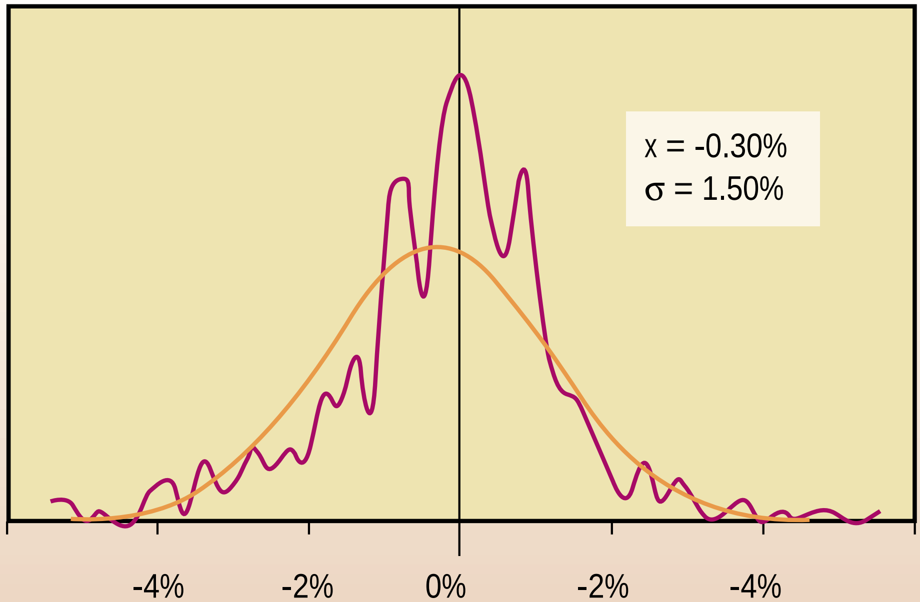
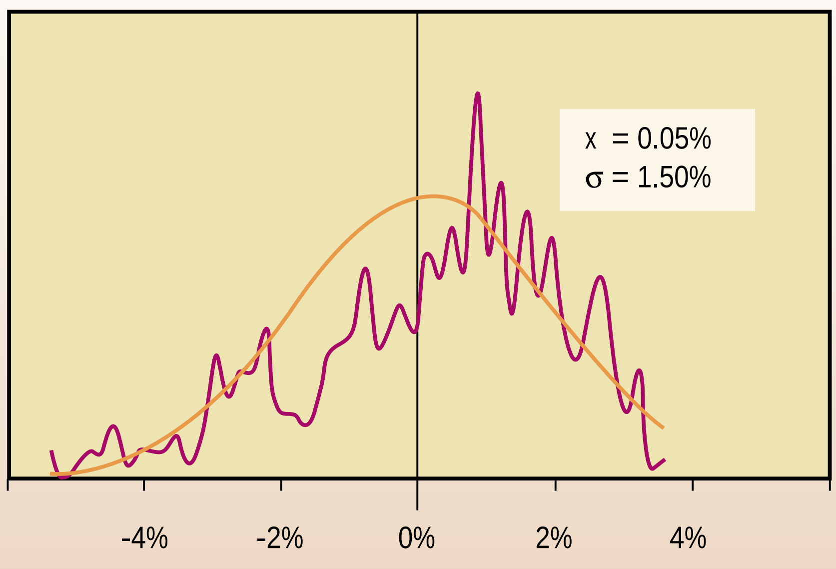


Figure 8. Distribution of differences between predicted and measured values for stripline.



and comparisons of the electrically measured and the theoretically calculated results are presented in Figure 8 and Figure 9.

Figure 9. Distribution of differences between predicted and measured values for embedded microstrip



Discussion

Accuracy of the electrical measurements is estimated at 1% to 2%. This depends upon the impedance value and the quality of the interconnection between the test equipment and the test sample. Test samples were designed to be electrically balanced, but the manufacturing process will obviously not produce perfectly balanced traces.

Microsection dimensions have an estimated accuracy of 1%; however, the model assumes symmetry and this will introduce a further small averaging error estimated at 1%. The total uncertainty in the experimental results is, therefore, estimated at 3% to 4%. Figure 8 and Figure 9 show mean deviations of less than 0.5% with standard deviations of less than 2%.

These practical results clearly show that the differences between the measured electrical results and the numerically calculated results are well within the estimated uncertainty of the measurement method.

Conclusion

The authors have shown that the early methods for calculating controlled impedance can now be used on desktop PC's. The accuracy is as good as, if not better than, the published algebraic equations. The processing times are less than 10s, which is acceptable in most cases.

Furthermore, the number of configurations can be extended and trade cross-sectional profiles can be readily incorporated.

References

1. Wadell, Brian C., *Transmission Line Design Handbook*, Artech House 1991.
2. IPC-2141: Controlled Impedance Circuit Boards and High-Speed Logic Design, April 1996.
3. Cohn, Seymour B., "Characteristic Impedance of the Shielded-Strip Transmission Line," IRE Trans MTT, 2 July 1954 pp. 52–57.
4. Abramowitz, Milton and Irene A Stegun, *Handbook of Mathematical Functions*, Dover, New York, 1965.
5. Hilberg, Wolfgang, "From Approximations to Exact Relations for Characteristic Impedances," IEE Trans MTT-17, No. 5, May 1969 pp. 259–265.
6. Hart, Bryan, *Digital Signal Transmission*, Pub: Chapman and Hall 1988.
7. Harrington, Roger F., *Field Computation by Moment Methods*, Pub: MacMillan 1968.
8. CITS25, Differential Controlled Impedance Calculator, Polar Instruments Ltd., http://www.polar.co.uk, 1998.
9. Sadiku, Matthew N.O., *Numerical Techniques in Electromagnetics*, Pub: CRC Press 1992.
10. Silvester, P.P., "Microwave Properties of Microstrip Transmission Lines," IEE Proc. Vol. 115, No. 1, January 1969, pp. 43–48.
11. Silvester P.P. & R.L. Ferrari, *Finite Element for Electrical Engineers*, Pub: Cambridge University Press, 1983.
12. Brebbia, C.A., *The Boundary Element Method for Engineers*, Pub: Pentech Press, 1980.
13. Paris, Federico and Jose Canas, *Boundary Element Method: Fundamentals and Applications*, Pub: Oxford University Press, 1997.
14. Kobayashi, Masanori, "Analysis of the Microstrip and the Electro-Optic Light Modulator," IEEE Trans MTT–26, No. 2, February 1979, pp. 119–127.
15. Bogatin, Eric; Mike Justice, Todd DeRego, and Steve Zimmer, "Field Solvers and PCB Stack-up Analysis: Comparing Measurements and Modelling," IPC Printed Circuit Expo 1998 paper, 505-3.
16. Li, Keren and Yoichi Fujii, "Indirect Boundary Element Method Applied to Generalized Microstrip Analysis with Applications to Side-Proximity Effect in MMICs," IEE Trans. MTT–40, No. 2, February 1992, pp. 237–244.

The authors wish to acknowledge the assistance of Kemitron Technologies plc., Stevenage Circuits Ltd. and Zlin Electronics Ltd.

Surface microstrip results were yet to be completed at the submission date for this paper.

This paper originally appeared in *Circuit World*, Vol. 25, No. 4, October 1999. It is reprinted here with permission.

射 频 和 天 线 设 计 培 训 课 程 推 荐

易迪拓培训(www.edatop.com)由数名来自于研发第一线的资深工程师发起成立，致力并专注于微波、射频、天线设计研发人才的培养；我们于 2006 年整合合并微波 EDA 网(www.mweda.com)，现已发展成为国内最大的微波射频和天线设计人才培养基地，成功推出多套微波射频以及天线设计经典培训课程和 ADS、HFSS 等专业软件使用培训课程，广受客户好评；并先后与人民邮电出版社、电子工业出版社合作出版了多本专业图书，帮助数万名工程师提升了专业技术能力。客户遍布中兴通讯、研通高频、埃威航电、国人通信等多家国内知名公司，以及台湾工业技术研究院、永业科技、全一电子等多家台湾地区企业。

易迪拓培训课程列表：http://www.edatop.com/peixun/rfe/129.html

射频工程师养成培训课程套装

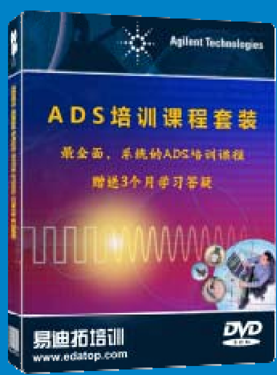
该套装精选了射频专业基础培训课程、射频仿真设计培训课程和射频电路测量培训课程三个类别共 30 门视频培训课程和 3 本图书教材；旨在引领学员全面学习一个射频工程师需要熟悉、理解和掌握的专业知识和研发设计能力。通过套装的学习，能够让学员完全达到和胜任一个合格的射频工程师的要求…

课程网址：http://www.edatop.com/peixun/rfe/110.html

ADS 学习培训课程套装

该套装是迄今国内最全面、最权威的 ADS 培训教程，共包含 10 门 ADS 学习培训课程。课程是由具有多年 ADS 使用经验的微波射频与通信系统设计领域资深专家讲解，并多结合设计实例，由浅入深、详细而又全面地讲解了 ADS 在微波射频电路设计、通信系统设计和电磁仿真设计方面的内容。能让您在最短的时间内学会使用 ADS，迅速提升个人技术能力，把 ADS 真正应用到实际研发工作中去，成为 ADS 设计专家...

课程网址： http://www.edatop.com/peixun/ads/13.html

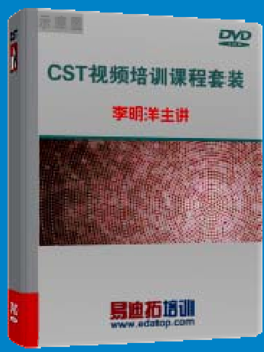


HFSS 学习培训课程套装

该套课程套装包含了本站全部 HFSS 培训课程，是迄今国内最全面、最专业的 HFSS 培训教程套装，可以帮助您从零开始，全面深入学习 HFSS 的各项功能和在多个方面的工程应用。购买套装，更可超值赠送 3 个月免费学习答疑，随时解答您学习过程中遇到的棘手问题，让您的 HFSS 学习更加轻松顺畅…

课程网址：http://www.edatop.com/peixun/hfss/11.html

CST 学习培训课程套装



该培训套装由易迪拓培训联合微波 EDA 网共同推出，是最全面、系统、专业的 CST 微波工作室培训课程套装，所有课程都由经验丰富的专家授课，视频教学，可以帮助您从零开始，全面系统地学习 CST 微波工作的各项功能及其在微波射频、天线设计等领域的设计应用。且购买该套装，还可超值赠送 3 个月免费学习答疑…

课程网址：http://www.edatop.com/peixun/cst/24.html

HFSS 天线设计培训课程套装

套装包含 6 门视频课程和 1 本图书，课程从基础讲起，内容由浅入深，理论介绍和实际操作讲解相结合，全面系统的讲解了 HFSS 天线设计的全过程。是国内最全面、最专业的 HFSS 天线设计课程，可以帮助您快速学习掌握如何使用 HFSS 设计天线，让天线设计不再难…

课程网址：http://www.edatop.com/peixun/hfss/122.html

13.56MHz NFC/RFID 线圈天线设计培训课程套装

套装包含 4 门视频培训课程，培训将 13.56MHz 线圈天线设计原理和仿真设计实践相结合，全面系统地讲解了 13.56MHz 线圈天线的工作原理、设计方法、设计考量以及使用 HFSS 和 CST 仿真分析线圈天线的具体操作，同时还介绍了 13.56MHz 线圈天线匹配电路的设计和调试。通过该套课程的学习，可以帮助您快速学习掌握 13.56MHz 线圈天线及其匹配电路的原理、设计和调试…

详情浏览：http://www.edatop.com/peixun/antenna/116.html

我们的课程优势：

※ 成立于 2004 年，10 多年丰富的行业经验，

※ 一直致力并专注于微波射频和天线设计工程师的培养，更了解该行业对人才的要求

※ 经验丰富的一线资深工程师讲授，结合实际工程案例，直观、实用、易学

联系我们：

※ 易迪拓培训官网：http://www.edatop.com

※ 微波 EDA 网：http://www.mweda.com

※ 官方淘宝店：http://shop36920890.taobao.com